

Degeneracy Breakdown as a Source of Supernovae Ia

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February 25, 2018

This is an author-created, un-copyedited version of an article published in *Journ. of Cosmol. & Astropart. Phys.*1801 (2018) no.01, 052.

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Abstract

In a confined system of multiple Fermions, the particles are forced into high energy levels by the Pauli Exclusion Principle. We refer to this system as a Pauli tower. We pursue the investigation of a model for sub-Chandrasekhar supernovae Ia explosions (SNIa) in which the energy stored in the Pauli tower is released to trigger a nuclear deflagration. The simplest physical model for such a degeneracy breakdown and collapse of the Pauli tower is a phase transition to an exactly supersymmetric state in which the scalar partners of protons, neutrons, and leptons become degenerate with the familiar fermions of our world as in the supersymmetric standard model with susy breaking parameters relaxed to zero. We focus on the ability of the susy phase transition model to fit the total SNIa rate as well as the delay time distribution of SNIa after the birth of a progenitor white dwarf. We also study the ejected mass distribution and its correlation with delay time. Finally, we discuss the expected SNIa remnant in the form of a black hole of roughly Jupiter mass and the prospects for detecting such remnants.

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1 Introduction

In the 1930's S. Chandrasekhar [1] famously showed that electron degeneracy pressure would make certain stars now known as white dwarfs classically stable up to about 1.4 solar masses. Slightly below this mass spontaneous nuclear fusion would erupt to destabilize the star. In 1973 it was proposed [2],[3] that mass accretion[†] onto white dwarfs from a binary partner could cause this Chandrasekhar mass to be approached from below at which point nuclear fusion would take over leading to an explosion which could be identified as a type Ia supernova. The clear evidence of fusion by-products expected from a Carbon or Oxygen White Dwarf progenitor supports the Whelan-Iben idea. In addition, the prediction that supernovae occur at the Chandrasekhar mass has suggested a possible explanation for the uniformity of the events. [‡] Since the seventies most of the theoretical work on supernovae has been based on transfer of matter onto white dwarf stars from a non-degenerate star or coalescence of a white dwarf star and another degenerate star. The great diversity of such ideas that have been proposed to resolve the supernova puzzles means that it might be no longer useful to refer to these as a “standard model”. Instead we refer to all these ideas collectively as “binary models”. For a comprehensive recent review see ref. [5]. Now, more than four decades later, a consensus in the astronomy community on the specific initial state and a quantitative understanding of the subsequent explosion has been elusive. In addition, there is now strong evidence for a sub-Chandrasekhar component [6],[7] which, in some models, weakens the connection with the Chandrasekhar mass.

Within the binary models for SNIa there is a distinction between the single degenerate (SD) scenario in which there is a nuclear burning star donating matter to the degenerate white dwarf and the double degenerate (DD) scenario in which a second degenerate white dwarf donates to a primary, more massive white dwarf progenitor. Both mechanisms and their variations are subject to a host of problems as discussed in a number of old and recent reviews. In addition to [5] we can cite [8] and [9]. In particular both mechanisms when faced with observational constraints underestimate the SNIa rate and rely on unverified assumptions about the frequency of the requisite initial state. Historically, when a phenomenon resists explanation in a standard model for multiple decades, the resolution of the puzzle often requires some radical new physics input.

Up until some five years ago the DD scenario was strongly disfavored and the obstacles to a satisfying theory based on this scenario were summarized as follows in the still cogent 2000 Review by Hillebrandt and Niemeyer [10]:

“Besides the lack of convincing direct observational evidence for sufficiently many appro-

[†]In this paper we use the term “accretion” as synonymous with mass growth regardless of the source. An isolated or non-interacting star would have zero accretion rate. There could also be some small accretion from the interstellar medium.

[‡]It should be noted, however, that, in addition to the “normal” supernovae of prime cosmological importance, a substantial fraction of SNe are quite diverse and do not exhibit this uniformity [4].

appropriate binary systems, the homogeneity of ‘typical’ SNe Ia may be an argument against this class of progenitors. It is not easy to see how the merging of two white dwarfs of (likely) different mass, composition, and angular momentum with different impact parameters, etc, will always lead to the same burning conditions and, therefore, the production of a nearly equal amount of ^{56}Ni .”

On the other hand, the SD scenario, although conceptually easier to envision, comes with its own set of puzzles. For instance there is no evidence of a binary partner remnant [11], [12] and no evidence of significant absorption by the partner.

The dilemma was heightened in 2010 with the observation that accretion in the SD scenario would necessarily be accompanied by significant X-ray emission and observed X-ray activity was far below what would be required if the entire SNIa rate was to be understood as accretion onto white dwarfs several tenths of a solar mass below the Chandrasekhar mass. Quantitatively, whereas an average accretion rate of $10^{-7} M_{\odot}/yr$ would be required to bring $1.2 M_{\odot}$ dwarfs to the Chandrasekhar mass at a sufficient rate, no more than one to two percent of this accretion rate is consistent with observations [13] [14]. Recently, an analogous constraint on the DD scenario has been proposed based on the absence of expected polarization in supernova light [15]. Nevertheless and in spite of the strong criticism of ref. [10], most but not all current theoretical work is based on the DD scenario as in ref. [16]. The simulations in this latter work were, in fact, quite influential in redirecting attention to the DD progenitors.

The purpose of this article is to refine the analysis of the susy phase transition model for SNIa [17]. In this model every white dwarf, whether in a binary system or not, has a characteristic mass-dependent lifetime. Most white dwarfs have a lifetime longer than the current age of the universe but those in a certain sub-Chandrasekhar mass range can have lifetimes in the tenths of a Gigayear range. Unless made explicit, masses in this article are defined relative to the solar mass, M_{\odot} .

The primary parameters of this model were a critical matter density, ρ_c , and a minimum lifetime, τ_0 . We generalize the model via a third free parameter, b_0 , to allow the possibility of a phase transition suppression at high pressure analogous to that observed in a superheated liquid. Finally, we assume, as a fourth parameter, an average white dwarf accretion rate parametrized by some c_0 which causes white dwarfs to increase in mass up to that at which a significant fraction of the white dwarfs undergo the supernova phase transition. This parameter is required to be consistent with the above-mentioned limits on the accretion in binary systems of a white dwarf with a main sequence partner.

The number of these parameters is comparable to the minimum number in the binary model (mean and width of the binary mass distributions and the accretion rate distribution or initial separation distribution even treated as average values). Clearly no prediction of the supernova rate and delay time distributions in the binary models can be made without knowing or assuming something about initial state distributions and then they can be tested against present or future observations. In the present paper we assume the accretion rate

distribution can be adequately replaced by an average value. At present no known binary systems are clearly thermonuclear supernova candidates. In the phase transition model, the supernova rates and delay time distribution are insensitive to the presence of binary partners and are related to known white dwarf mass distributions. In recent years improved analyses have been made of the delay time distribution of SNIa. It is found that many supernovae occur within a few tenths of a Gyr after the birth of the progenitor white dwarf but there are also many occurring after Gyr delay times.

In the framework of the phase transition model we are able to quantitatively fit the total SNIa rate and the rates in three recently observed delay time bins, The ejected mass distribution is also predicted and a sharp bimodal peaking at short delay times is suggested.

The most recent high statistics data release from the Sloan Digital Sky Survey (SDSS) contains a clean sub-sample in which the mass of the white dwarfs and their age from birth as a white dwarf are reasonably well determined [18]. This sample shown in figure.1 contains 95 DA type white dwarfs which have a thin hydrogen atmosphere (about 10^{-5} of the total mass) and 55 DB type white dwarfs which have a similarly thin helium atmosphere. In this paper we focus on the DA white dwarfs.

The organization of this paper is as follows. In Section 2, we start with the mass distribution of hot white dwarfs which should approximate the distribution at birth. We then compare the mass distribution of the old white dwarfs with known ages [18]. This allows us to estimate the average accretion rate.

In Section 3 we briefly review the theory of the phase transition to exact susy from [17] and discuss the possibility of extending the model to include a transition-suppressing pressure term in the action. The analog would be the suppression of the boiling transition in a liquid under high pressure. Conservation of degrees of freedom in this model requires the existence of a broken susy in our world although the masses in this phase can be quite high consistent both with their non-observation in current accelerator searches and with susy grand unification theory.

In Section 4 we describe a four parameter monte carlo and discuss how the parameters are constrained by 1σ fits to the observed delay time distribution.

In Section 5 we discuss the ejected mass distribution and its correlation with delay times.

In Section 6 we discuss the predicted supernova remnants and their eluding of current searches as well as some consequences for future searches.

Section 7 is reserved for a summary and discussion of results.

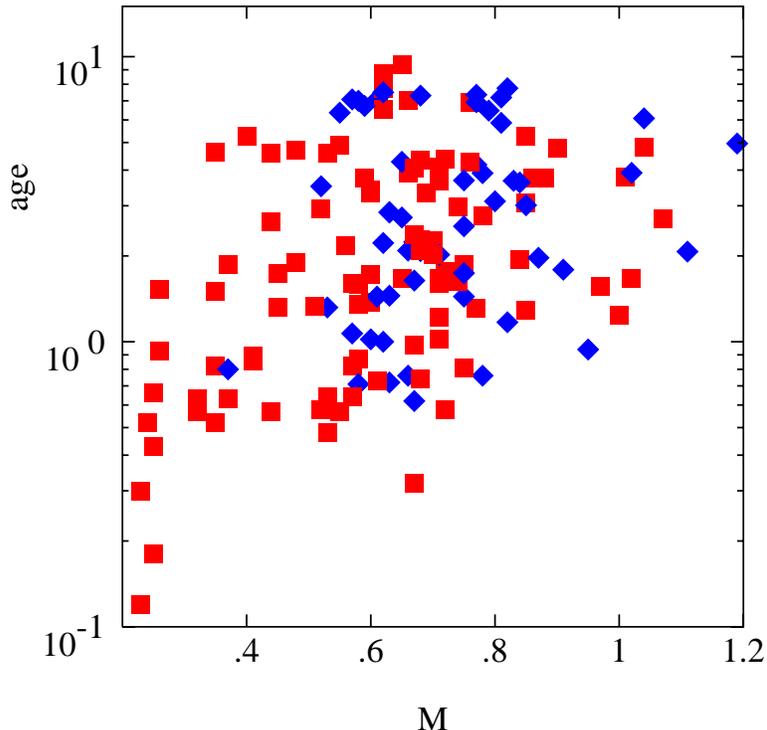


Figure 1: The age in Gyr versus mass in a clean sample of old white dwarfs. DA dwarfs are shown in red squares while DB dwarfs are shown in blue diamonds.

2 White dwarf mass distributions and accretion

The white dwarf initial mass function $F(M(0))$ above the peak at $0.6 M_{\odot}$ is obtained from the Salpeter initial mass function for a main sequence star followed by an initial to final mass function which linearly relates the white dwarf production rate at birth mass, $M(0)$, to that of the parent star. The relation

$$\frac{dN}{dM(0)} = a_0 F(M(0)) = a_0 ((M(0) - 0.478)/0.09028)^{-2.35} \quad m(0) > 0.62 M_{\odot} \quad (2.1)$$

gives a good fit to the decreasing part of the hot white dwarf mass distribution as shown in figure 2, [19]. The curve shown corresponds to normalization $a_0 = 1000$ for the observed sample in that plot. a_0 is not to be confused with its use as the initial orbital separation in a binary system. We will take this distribution to represent that of white dwarfs at birth ($t=0$) At later times we will assume an average mass increase due to accretion corresponding

to the form

$$M(t) = M(0) + c_0 t \quad (2.2)$$

with c_0 being a free parameter in the monte carlo. Of course, particular stars could accrete at a lower or higher rate and we assume that an average accretion rate is a useful first approximation.

We could phenomenologically extend the fit down to $0.35M_\odot$ with the form

$$\frac{dN}{dM(0)} = F(M(0)) = 0.474a_0/(1 + (M(0) - .62)^2/.048)^2 \quad 0.35 < M(0) < 0.62 \quad (2.3)$$

However, in our monte carlo we find that no white dwarf with a birth mass less than 0.8 results in a supernova in less than a Hubble time so we can ignore eq.2.3.

The current parameters of the susy model assume that WD's of mass greater than 0.9 in DD binaries excluded from the data of figure.2 should not be produced in numbers significantly greater than the Salpeter IMF-based numbers of eq. 2.1. Current data is not in disagreement with this assumption.

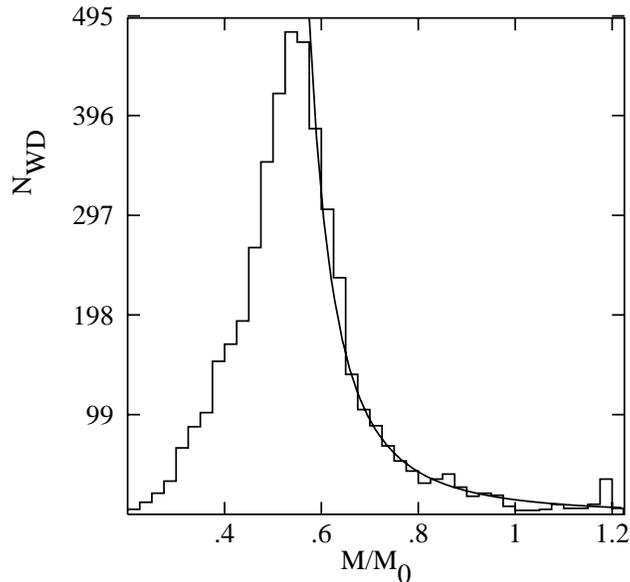


Figure 2: The SLOAN sample of hot ($T_{eff} > 12000K$) DA white dwarfs [19],[20] compared to the theoretical fit from eq. 2.1 in the high mass tail.

The SLOAN data shown in figure.2 indicate that a fraction $f = 0.292$ have masses above $0.6 M_{\odot}$. We normalize to a sample of 10^{10} white dwarfs as in a Milky Way-like galaxy following the hot white dwarf mass distribution shown in figure.2. We assume that the birth rate of white dwarfs has been constant for 12.8 Gyr and is proportional to this hot white dwarf mass distribution. That is, with a new a_0 normalized to this prototype galaxy,

$$\frac{d^2 N_{WD}}{dt dM(0)} = a_0 F(M(0)) \quad (2.4)$$

with

$$f \cdot 10^{10} = a_0 12.8 \text{Gyr} \int_{0.6}^{1.4} dM(0) F(M(0)) \quad (2.5)$$

so that

$$a_0 = 5.48 \text{WD/yr}/10^{10}\text{WD} \quad . \quad (2.6)$$

Here we have assumed that, at first order, the accretion rate is small enough to be ignored as is the white dwarf depletion due to supernovae.

As a rough measure of the accretion rate we take the 95 cool DA white dwarfs with measured masses and ages from ref.[18]. We assume that some fraction f_0 are isolated and therefore do not accrete and that the the others have accreted at an average rate c_0 . Under these assumptions Eq. 2.2 then statistically defines a birth mass. The peak of the hot white dwarf mass distribution comes from solar mass stars that have a long lifetime on the main sequence. The high mass tail could, however, reveal the scale of accretion leading to the cool white dwarf spectrum. We scan over values of f_0 and c_0 seeking the least χ^2 in comparison with figure.2 and eq.2.1. In table1 we plot the best fit predicted numbers at birth $N(M(0))$ to compare with eq.2.1 normalized at a mass of 0.7. The procedure is to extrapolate each cool white dwarf of given mass and age back to its birth mass assuming an average accretion rate. One then tabulates the birth mass distribution to compare with the observed distribution of hot white dwarfs, calculating the best fit f_0 and c_0 .

The resulting best fit estimates are

$$f_0 = 0.34 \quad c_0 = 4 \cdot 10^{-12} M_{\odot}/\text{yr} \quad . \quad (2.7)$$

The average accretion rate is independently determined below in a monte carlo of the phase transition model. In the Monte Carlo, however, we treat all white dwarfs as accreting with an average rate c_0 which turns out to be very small partially because most white dwarfs are isolated. The exercise of table1 is taken as a crude consistency check and the numbers in eq. 2.7 are not used in later sections. Above a birthmass of 1.1 the numbers in table 1 predicted from the cool white dwarf data are significantly lower than the initial mass function which could be taken as an indication that, due to supernovae, a significant number of white dwarfs originally at mass of 1.1 or above do not survive cooling to low temperatures due to supernovae.

M(0)	N(M(0))	eq. 2.1
0.5	10	11.8
0.6	20	19.7
0.7	26	26.0
0.8	6	10.05
0.9	3	5.75
1.0	5	3.49
1.1	1	2.31
1.2	0	1.63
1.3	0	1.20

Table 1: Best fit numbers of hot white dwarfs $N(M(0))$ compared to the observed numbers from eq. 2.1 assuming a fraction f_0 is isolated (non-accreting) and that the others accrete with average rate c_0 .

3 Degeneracy breakdown in dense matter

We briefly review here the model of [17] for the release of the energy stored in dense matter due to the Pauli exclusion principle. Further details can be found in that reference.

Earlier papers on the susy phase transition in the context of gamma ray bursts were refs.[21], [22], [23], [24] which estimated an energy release per unit mass of 0.02.

In the string landscape picture, the universe can exist in a large number of local minima of the vacuum energy. The transition probability per unit space time volume from one local minimum to another is given in the thin wall approximation by the vacuum decay formula [25]

$$\frac{d^3 P}{dt d^3 r} = A e^{-B(r)} \quad (3.8)$$

where the vacuum decay action, B , is proportional to the inverse cube of the difference between the vacuum energies. One would expect that, in the presence of matter, this action is proportional to the inverse cube of the total energy density difference. In dense stars the vacuum energy density is negligible compared to the matter density difference. As discussed in ref.[17], the matter density difference is equal to the Pauli energy. In the Fermi gas model, this energy stored in the Pauli towers is proportional to the matter density, $\rho(r)$, so the action $B(r)$, in first approximation, can be written

$$B(r) = \left(\frac{\rho_c}{\rho(r)}\right)^3 \quad (3.9)$$

where the critical density ρ_c , treated as a free parameter, is related to the surface tension of the bubble wall between the two phases. In a white dwarf star with Chandrasekhar density

profile $\rho(r)$, the transition probability per unit time is

$$\frac{dP}{dt} = \frac{1}{\tau_0 V_0} \int d^3r e^{-B(r)} \equiv \frac{1}{\tau} \quad . \quad (3.10)$$

This equation defines the lifetime, τ of each white dwarf as a function of its mass. Without loss of generality one can choose V_0 such that the free parameter, τ_0 , is the minimum lifetime over all white dwarfs. Thus there are two primary free parameters, a critical density ρ_c and a minimum lifetime, τ_0 .

The Chandrasekhar density profile is defined by the balance between the gravitational pressure gradient

$$\frac{dP_G}{dr} = -\rho(r)GM(r)/r^2 \quad (3.11)$$

and the degeneracy pressure gradient

$$\frac{dP_D}{dr} = \frac{ab}{3} \rho^{-2/3} \frac{x^4}{1+x^2} \frac{d\rho}{dr} \quad (3.12)$$

where a is proportional to the classical electron energy density

$$a = \frac{m_e^4 c^5}{3\pi^2 \hbar^3} = \frac{m_e c^2}{3\pi^2 r_0^3} \quad , \quad (3.13)$$

r_0 being the classical radius of the electron,

$$b = \frac{\pi \hbar}{m_e c} \left(\frac{3}{2\pi m_N} \right)^{1/3} \quad , \quad (3.14)$$

with m_N being the nucleon mass and

$$x = b\rho^{1/3} \quad . \quad (3.15)$$

Using Chandrasekhar's calculation one finds that for low mass dwarfs, the density is small leading to a large lifetime. For masses approaching the Chandrasekhar mass ($\approx 1.4M_\odot$) the volume approaches zero and the density approaches infinity leading again to a large lifetime from eq. 3.10. Consequently, some white dwarfs very close to the Chandrasekhar mass could be long-lived while slightly lighter dwarfs would rapidly decay. The same effect could come from a pressure term in the action as discussed below. Thus the lifetime, τ , is expected to be a quasi-parabolic function of white dwarf mass.

Of course one might expect that there are sub-leading corrections to the Coleman-DeLuccia formula. This formula gives the probability per unit space time volume to nucleate the first critical bubble of true vacuum. If this probability is large the initial nucleation is followed rapidly by further bubble creation.

One could also ask whether there are transition suppressing pressure terms in the action increasing with density such as in the case of a super-heated liquid. Thus, with this analogy, one could allow the possibility that the action is enhanced at high density parametrized by a b_0 :

$$B(r) = \left(\frac{\rho_c}{\rho(r)}\right)^3 + b_0(\rho(r)/\rho_c)^{4/3} \quad . \quad (3.16)$$

Such a pressure term would enlarge the region of validity of the Coleman-DeLuccia formula since the latter is dependent on the action being large. For a particular choice of parameters chosen from among those preferred in the Monte Carlo of 4, the lifetime as a function of white dwarf mass is shown in figure.3 which is also useful in that it gives an overview of the salient features of the phase transition model.

White dwarfs are born at zero age and, neglecting accretion, grow vertically upward in the plot of figure.3. The survival probability to age t is then

$$P_s = e^{-t/\tau(M)} \quad . \quad (3.17)$$

In the absence of accretion and with the given parameter choice one would expect to find no old white dwarfs in the quasi-parabolic region. This forbidden region moves to the right and is squeezed against the Chandrasekhar mass as the critical density, ρ_c is increased. The region also moves vertically upward if the minimum lifetime, τ_0 is increased.

4 The four parameter monte carlo

In fitting the observational data on SNIa we scan over values of the four parameters $\rho_c, \tau_0, b_0,$ and c_0 . In the presence of accretion, the transition probability per unit time becomes

$$\frac{dP}{dt} = \frac{1}{\tau_0 V_0} \int d^3r e^{-B(r)} \equiv \frac{1}{\tau(M)} \quad (4.18)$$

where the white dwarf mass M is related to the birth mass by eq.2.2. The survival probability to age t then becomes

$$P_s(t) = e^{-\int_0^t dt'/\tau(M(t'))} \quad . \quad (4.19)$$

In the absence of accretion, M is constant and the survival probability reduces to eq. 3.17. Old white dwarfs in the parabolic region should be those that are relatively rapidly accreting which might be susceptible to observational test.

In the case of no accretion (e.g. solitary white dwarfs) the stars age vertically upwards in figure.3. The probability of surviving until reaching the $\tau(M)$ curve is then $1/e$. With

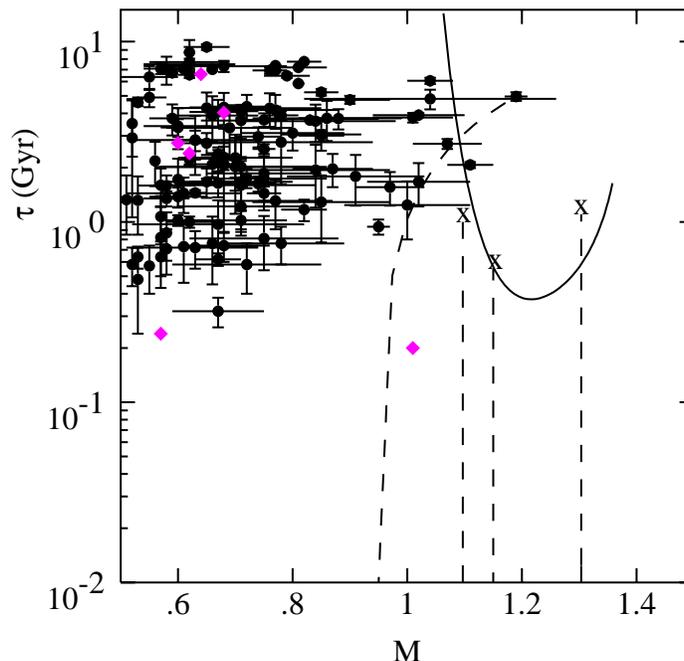


Figure 3: The white dwarf lifetime in Gyr as a function of mass is shown by the quasi-parabolic curve using, as examples, parameters $\rho_c = 8.14 \cdot 10^7 g/cm^3$, $\tau_0 = 0.372$ Gyr, and $b_0 = 0.87$. The data on white dwarf masses and ages from ref. [18] are shown in black while the six nearest white dwarfs are shown in magenta diamonds. The vertical dashed lines exemplify white dwarfs aging without accretion until producing a supernova at the delay times indicated by “x”. The curved dashed line shows a possible trajectory of the highest mass DA dwarf in the data of ref. [18] corresponding to an assumed accretion rate of $4.8 \cdot 10^{-11} M_\odot/yr$. With the chosen model parameters, this white dwarf would be unlikely to survive to its measured age without accretion but the chosen hypothetical accretion rate allows it to survive by spending little time in the quasi-parabolic region.

the indicated critical density, the two shown high mass DA dwarfs from the Bergeron et al. sample [18] are unlikely to survive to the observed age without accretion but can have a high survival probability if they are accreting and age along the curved path. Alternatively, the critical density parameter can be increased which results in the parabola moving to the right. Sirius b, the right-most magenta diamond, is unlikely to produce a supernova unless the critical density is significantly lower than the value taken in figure. 3. In this improbable case many of the high mass dwarfs from ref. [18] would have to be undergoing above average

accretion in order to survive to the measured ages.

Neglecting the fall-off of $F(M(0))$ in the region of $M \approx 1.2$, the greatest supernova rate comes from the minimum of the $\tau(M)$ curve. White dwarfs with masses above and below the minimum have roughly equal intermediate lifetimes leading to a double peak in the ejected mass distribution at these intermediate delay times.

The probability to survive to age t and then make the supernova phase transition in the next interval dt is

$$dP(t) = P_s(t) dt / \tau(M(t)) \quad . \quad (4.20)$$

The energy released in the phase transition may be significantly greater than the energy released afterwards by fusion which results in the star being totally disrupted except for a very small remnant discussed in Section 6. The supernova distribution as a function of ejected mass, $M(t)$ and delay time t is therefore,

$$\frac{d^2 N_{SN}}{dM(0)dt} = a_0 F(M(0)) P_s(t) / \tau(M(t)) \quad (4.21)$$

with

$$M(t) = M(0) + c_0 t \quad . \quad (4.22)$$

The situation with respect to delay times is well summarized in the graph of ref [26] reproduced here in figure. 4. In refs. [27] and [26] the supernova rates were divided into three delay time bins bounded by the delay times:

$$t_1 = 0.04Gyr \quad t_2 = 0.4Gyr \quad t_3 = 2.4Gyr \quad t_4 = 13.8Gyr \quad . \quad (4.23)$$

The average supernova rate in the three bins is

$$S_i = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} dt \frac{dN_{SN}}{dt} \quad . \quad (4.24)$$

The recovered delay times [26] with the smallest statistical errors are shown in filled red circles in this graph. Previous measurements [27] with larger statistical errors are shown in open blue squares shifted slightly to the left for clarity. Measurements from 2010 [28] are shown in open black circles. These latter, unfortunately, are in statistical disagreement with the later measurements implying one or more methods are subject to biases or large systematic errors.

From the graph of figure. 4 and the data of ref. [26] we read the 1σ SNIa rates in the three bins in units of SNIa/yr per Gyr delay time per prototype galaxy of 10^{10} white dwarfs:

$$\begin{aligned} 1.11 \cdot 10^{-2} &< S_1 < 1.71 \cdot 10^{-2} \\ 1.88 \cdot 10^{-3} &< S_2 < 3.16 \cdot 10^{-3} \\ 1.44 \cdot 10^{-4} &< S_3 < 2.33 \cdot 10^{-4} \end{aligned} \quad (4.25)$$

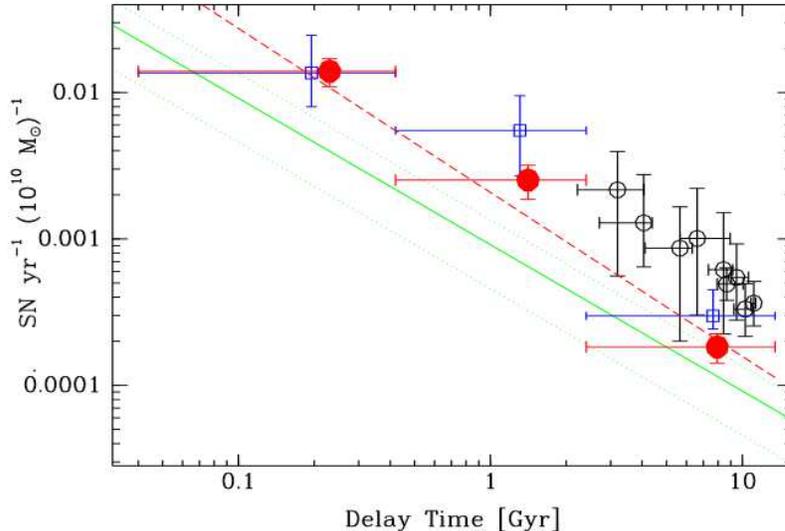


Figure 4: The SNIa rate as a function of delay time from [26]. See text. The key point is that the greatest supernova rate per unit delay time is in the lowest bin (less than 0.4 Gyr) but the delay time distribution from white dwarf birth extends into the multi-Gyr range.

or

$$\begin{aligned}
 0.135 < R_{21} \equiv S_2/S_1 < 0.227 \\
 0.008 < R_{31} \equiv S_3/S_1 < 0.021
 \end{aligned}
 \tag{4.26}$$

The approximate linearity of the curve joining the filled red points in figure.4 with a power of approximately -1 ($p = -1.12 \pm 0.08$) has been interpreted [26] as supporting the DD scenario providing the initial separation distribution of the two white dwarfs is approximately d^{-1} . This power law fit requires a sharp cutoff at low delay times [5] which introduces additional parameters. One can question whether the linear fit requires an implausibly high frequency of binary white dwarfs with a high combined mass. It would, of course, be desirable in the future to reduce the large bin size of the prompt events since this is where the various binary models discussed in refs. [5],[29] most disagree among themselves and with the prediction of the phase transition model.

If we assume in the DD scenario that an evolving star that would lead to a final mass between 1.35 and 1.45 produces with probability q a binary white dwarf system with that total mass, the birthrate (eq.2.4) of such a binary system and an upper limit to the supernova rate should, from the Salpeter initial mass function, be

$$dN/dt = a_0 q \int_{1.35}^{1.45} F(M(0)) dM(0) = q \cdot 1.8 \cdot 10^{-3} \text{SN/yr}/10^{10} \text{WD} \quad . \tag{4.27}$$

Assuming q must be no more than unity this underpredicts the SNIa rate by a factor of about 10 as is confirmed in more detailed treatments. Thus investigators of the double

degenerate model must assume a large and as yet unobserved component of high mass binaries corresponding, effectively, to a value $q \approx 10$ in eq.4.27. The alternative path to a binary initial state of white dwarfs namely independent but simultaneous production of white dwarfs from binary main sequence stars is presumably less likely. In the clean sample of ref.[18] only 13% of white dwarfs are known or suspected to be in double degenerate configurations and, of these, the heaviest dwarf has mass of only 0.66. We recognize, however, that the supernova rate is not found to be the same across galaxy types. This is probably an important clue that has not as yet been addressed in the susy model so, at present, our rates should be considered as an average over galaxy types. It must also be noted that the delay time data of [26] includes statistical errors only and that systematic errors due to galaxy type effects are difficult to estimate. An effort to explain the delay time distribution as primarily a function of host galaxy types [30],[31] must await further data before quantitative results can be compared with those of the susy phase transition model.

Moreover, with respect to the linear fit, the required inverse d initial separation distribution is not strongly motivated from theory, finiteness requires that the linearity fails at small delay times [5], and in addition the filled red data points might show a slight negative curvature, i.e. the observed value of S_2 in figure. 4 is some two standard deviations higher than the best fit.

Any model that produces a curve going through the three S_i in figure.4 will imply a roughly correct total supernova rate but, to be physically meaningful, the model must show a connection to an adequate number of progenitors.

As we will see the delay time distribution can be adequately fit in the phase transition model and is naturally finite when integrated to zero delay time. In this model, the integral over delay times at fixed initial mass can be done analytically so that

$$S_i = \frac{1}{t_{i+1} - t_i} \int dM(0) \int_{t_i}^{t_{i+1}} \frac{d^2 N_{SN}}{dM(0)dt} dt = a_0 \int dM(0) F(M(0)) \frac{P_s(t_i) - P_s(t_{i+1})}{t_{i+1} - t_i} \quad (4.28)$$

The total supernova Ia rate is

$$R_{SN} = a_0 \int dM(0) F(M(0)) (1 - P_s(t_4)) \quad . \quad (4.29)$$

An advantage of the ratios R_{21} and R_{31} is that they are independent of the a_0 parameter, i.e. they should be the same for any sample of white dwarfs following the high mass part of the hot white dwarf distribution in figure.2 regardless of the total number of white dwarfs in the sample. To the extent that the DB dwarfs have the same initial mass function and a similar accretion rate their effect is included in the predicted supernova rates per 10^{10} white dwarfs. The phase transition probability would not differ between dwarfs with a thin atmosphere of hydrogen or helium providing their density profiles were largely the same. In the phase transition monte carlo we scan over the four parameters requiring that the

	min	variable	max
scan :	$5 \cdot 10^7 \text{ g/cm}^3$ 0.15 Gyr 0 0	ρ_c τ_0 b_0 c_0	$1.5 \cdot 10^8 \text{ g/cm}^3$ 0.45 Gyr 4 $10^{-11} M_\odot/\text{yr}$
require :	$1.11 \cdot 10^{-2}$ 0.135	S_1 R_{21}	$1.71 \cdot 10^{-2}$ 0.227
find :	$7.3 \cdot 10^7 \text{ g/cm}^3$ 0.18 Gyr 0.16 0 $1.7 \cdot 10^{-3}$ $1.1 \cdot 10^{-4}$ 0.009	ρ_c τ_0 b_0 c_0 S_2 S_3 R_{SN}	$1.45 \cdot 10^8 \text{ g/cm}^3$ 0.39 Gyr 4 $10^{-11} M_\odot/\text{yr}$ $3.2 \cdot 10^{-3}$ $9.3 \cdot 10^{-4}$ 0.023

Table 2: Four parameter monte carlo in phase transition model showing the range of scanned variables, the conditions required, and the resulting minimum and maximum values of the output parameters.

resulting S_1 and R_{21} fall within their 1σ ranges. The resulting ranges of the other quantities are then tabulated in table 2.

Fixing only S_1 and R_{21} the model predictions span the range of observations for S_3 and the total supernova rate. In the phase transition model the supernovae fall naturally into two classes, those occurring in isolated dwarfs and those occurring in binary systems with non-negligible accretion onto the white dwarf. The first class contains only those supernovae initiated by the vertical ageing in figure. 3 which would be expected to have a lower S_3 . The second class is represented by the curved path in figure. 3 and would have higher S_3 values. If the delay time recovery method is not biased the delay time distribution will be an appropriately weighted average of the two classes. In Table 3 we separate the results in the two classes.

The parameter space of the phase transition model could be further restricted if the above-noted tension between the long delay time measurements of [26] and [28] could be resolved. If the lower S_3 values are confirmed, the accretion rates favored by the Monte Carlo are only about 1% of those allowed by the x-ray data [13] which are themselves only about 1% of the rates that would be required if the full supernova Ia rate were to be explained in the single degenerate scenario. The prediction of low average accretion rates is significant since, with larger accretion rates, the absence of binary partner effects would be puzzling even in the phase transition model. The average accretion rate, c_0 , is an environmental parameter and not part of the phase transition theory. Excellent fits to the data can be found with c_0 fixed to be zero. In fact, with the preferred value found, the average white

require:	$0.011 < S_1 < 0.0171$ $0.135 < R_{21} < 0.226$	
scan:	$5 \cdot 10^7 \text{g/cm}^3 < \rho_c < 1.5 \cdot 10^8 \text{g/cm}^3$ $0.15 \text{Gyr} < \tau_0 < 0.45 \text{Gyr}$ $0 < b_0 < 4$ $0 < c_0 < 2 \cdot 10^{-12} M_\odot/\text{yr}$ $2 \cdot 10^{-12} M_\odot/\text{yr} < c_0 < 10^{-11} M_\odot/\text{yr}$	
find:	$0.246 \text{Gyr} < \tau_0 < 0.389 \text{Gyr}$	$0.184 \text{Gyr} < \tau_0 < 0.389 \text{Gyr}$
	$1.7 \cdot 10^{-3} < S_2 < 2.9 \cdot 10^{-3}$	$1.7 \cdot 10^{-3} < S_2 < 3.2 \cdot 10^{-3}$
	$1.1 \cdot 10^{-4} < S_3 < 2.6 \cdot 10^{-4}$	$1.9 \cdot 10^{-4} < S_3 < 9.3 \cdot 10^{-4}$
	$0.0068 < R_{31} < 0.0187$	$0.014 < R_{31} < 0.091$

Table 3: Four parameter monte carlo in phase transition model separated into the classes with smaller or larger accretion rates. As seen here a larger accretion rate leads to a larger S_3 as suggested in the data of ref. [28]. In both cases the accretion rates are far smaller than those that would be required in the SD scenario and that are ruled out by the x-ray data. Output variables that are insensitive to the accretion rates are not shown.

dwarf leading to a supernova does not grow significantly over its lifetime.

Instead of fixing S_1 and R_{21} as in table 1, one could just require an acceptable χ^2 fit to the three S_i :

$$\chi^2 = \sum_i (S_i - S_{i,0})^2 / \Delta_i^2 \quad . \quad (4.30)$$

The best fit, shown in figure 5, is found at $\rho_c = 7.42 \cdot 10^7 \text{g/cm}^3$, $\tau_0 = 0.401, \text{Gyr}$ $b_0 = 1.59$, and $c_0 = 0$. There is a broad minimum in the χ^2 distribution implying that many solutions with parameters in the range of table 2 are statistically of equal quality. The maximum accretion rate among solutions with $\chi^2/\text{dof} < 1$ is $4 \cdot 10^{-12} M_\odot/\text{yr}$ implying that the average white dwarf gains no more than $0.06 M_\odot$ in mass in a Hubble time, not enough to affect the delay time distribution except slightly at the highest delay times. There could, of course, be a few white dwarfs showing significantly greater accretion (or mass transfer) rates. However, an observed average much greater than predicted here would be a problem for the susy model.

In discussing the predictions that follow from the white dwarf distribution of figure. 2, we must caution that the data may be biased by selection effects against white dwarfs in binary systems. See, for example, the simulations of [32] for white dwarf + main sequence binaries. The phase transition model does not distinguish between white dwarfs in binary systems and isolated dwarfs. However, if the numbers of uncounted dwarfs from binary systems are large or their mass distribution deviates significantly from that of eq. 2.1 in the high mass tail, the parameters of the model will be affected. In extreme cases, the successes of the phase transition model might be called into question.

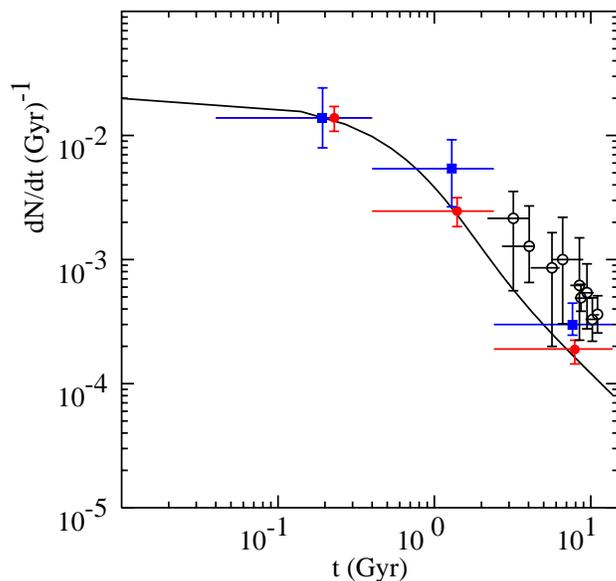


Figure 5: The SNIa rate as a function of delay time in the phase transition model with best fit parameters: $\rho_c = 7.42 \cdot 10^7 \text{g/cm}^3$, $\tau_0 = 0.401 \text{Gyr}$, $b_0 = 1.59$, and $c_0 = 0$. Data is taken from the graph of ref. [26] reprinted above in figure.4 .

In the case of zero average accretion rate, which is approximately true in all the low χ^2 solutions, the delay time distribution takes the simple form:

$$\frac{dN}{dt} = a_0 \int dm_0 F(m_0) \frac{e^{-t/\tau(m_0)}}{\tau(m_0)} . \quad (4.31)$$

5 The ejected mass distribution

Attempts to discover compact remnants of SNIa have up to now been unsuccessful suggesting the supernova process totally disrupts the white dwarf. This in turn implies in binary scenarios that the ejected mass should be very close to the Chandrasekhar mass. However, recent studies have shown ejected mass distributions extending down to $0.9 M_\odot$ [33]. That would imply compact remnants extending up to $0.5 M_\odot$. Numerous white dwarfs have been discovered in this mass range but never as supernova remnants. As noted in that reference the ejected mass distribution strongly suggests a sub-Chandrasekhar mechanism such as supplied by the present phase transition model.

From eq. 4.21 we can write the ejected mass distribution between delay times of t_i and t_{i+1} assuming the supernova totally disrupts the white dwarf as suggested by the absence of significant remnants and predicted in the present model apart from a small remnant to be discussed in the next section.

$$\frac{dN_i}{dM} = a_0 \int_{t_i}^{t_{i+1}} dt F(M - c_0 t) P_s(t) / \tau(M) \quad . \quad (5.32)$$

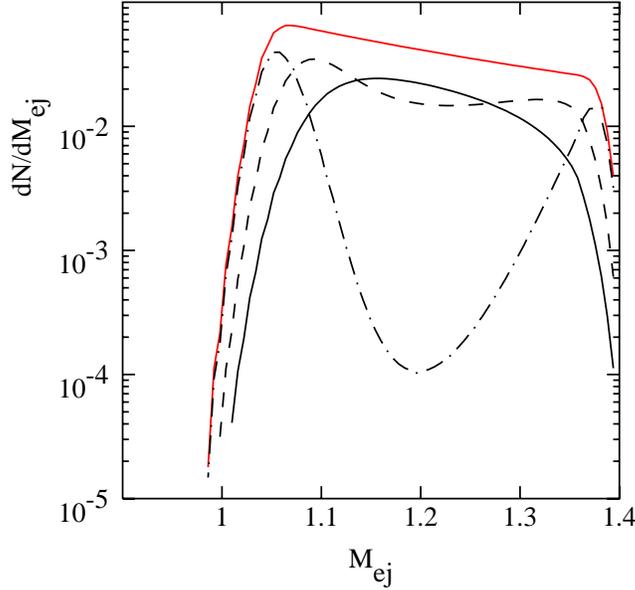


Figure 6: The ejected mass distributions for the susy model with parameters: $\rho_c = 7.42 \cdot 10^7 g/cm^3$, $\tau_0 = 0.401 \text{Gyr}$, $b_0 = 1.59$, $c_0 = 0.0$. The red curve gives the full ejected mass distribution. The solid black curve gives the mass distribution in the restricted delay time regions $t < 0.4 \text{Gyr}$ (solid black), $0.4 \text{Gyr} < t < 2.4 \text{Gyr}$ (dashed curve) and $2.4 \text{Gyr} < t < 13.8 \text{Gyr}$ (dash-dotted curve).

In the case $c_0 = 0$, the ejected mass distribution in the delay time interval between t_i and t_{i+1} can be written

$$\frac{dN_i}{dM} = a_0 F(M) (e^{-t_i/\tau(M)} - e^{-t_{i+1}/\tau(M)}) \quad . \quad (5.33)$$

These are plotted in figure 6 for the best fit parameters given in the previous section. The range of the ejected mass distribution agrees well with the data of ref. [33] but the possible

dip in the data is not reproduced by the model except in the case of moderate to late delay times.

One should note that the present model for sub-Chandrasekhar supernovae does not preclude a few percent of supernovae coming from accretion to the Chandrasekhar mass as is consistent with the x-ray data [13]. These events arise from white dwarfs punching through the low lifetime region due to much greater than average accretion or to white dwarfs born at high mass. The result would be a second peak in the ejected mass distribution as perhaps observed in the data analysis of ref. [33]. This is independent of the second peak expected in the current model at intermediate and late delay times as mentioned in section 4.

6 SNIa remnant in the Phase Transition Model

One of the great puzzles in the binary model of SNIa is the absence of detected compact remnants. In the phase transition model a small bubble of exact susy grows releasing Pauli energy until that energy plus the energy released through fusion is equal at least to the energy required to unbind the shell outside. If one treats the white dwarf as having the average density $3M/(4\pi R^3)$ the remnant has about 10^{-3} of the initial mass [17]. In a more precise treatment of the rapidly decreasing density with radius one might expect this 10^{-3} fraction to decrease. This remnant is predominantly made of the scalar partners of the protons, neutrons, and electrons. Since these cannot be supported by degeneracy pressure, the remnant collapses, after cooling, to a black hole of approximately Jupiter mass. Such a small black hole would be difficult to detect in the supernova aftermath. In this regard it is interesting to note the large number of Jupiter-like exoplanets that have been observed and a recent possible discovery of an ultralight compact remnant of a sub-luminous supernova [34]. In the typical galaxy there have been about 10^8 type Ia supernovae since the big bang so the phase transition model would predict this number of low mass black holes in the Milky Way. Charged particles falling into a black hole could be expected to radiate a few percent of their rest mass. This prediction could perhaps be tested in cosmic x-ray backgrounds [35],[36],[37]. In addition many stars could have captured one of these small black holes which might then be manifested by massive objects orbiting at large angle from the solar disks similar to the apparent “planet nine” in our own solar system. Such black hole planets might also be expected to have their own moons and/or accretion disks. The complete treatment of the growth of a susy bubble in a dense star and its remnant is a complicated dynamical calculation which has only been preliminarily considered in refs. [22],[23],[24].

7 Summary

The phase transition model has a number of advantages relative to the binary model explosion at the Chandrasekhar mass. The delay time distribution although not linear in a

log-log plot can easily fit the data. The ejected mass distribution naturally peaks below the Chandrasekhar mass and agrees with the observed range although the shape of the distribution departs from the current observations. The total supernova rate is related to known (primarily single white dwarf) mass distributions unlike the situation in the binary model which depends on unknown binary distributions and which, with plausible assumptions, underestimates the total rate. The phase transition model is insensitive to the numbers of binary systems unless the known mass distributions are greatly contaminated by unrecognized binary systems. Ideally, double degenerate systems are counted twice in the single WD mass distribution.

The absence of observed compact remnants which is challenging in the binary model is easily understood in the phase transition model which also makes an interesting prediction of remnant effects. The sphericity of supernova events and the absence of shadowing by a binary partner are predicted in the phase transition model with low accretion rates but remain puzzling in the binary model. In the DD scenario one must wonder why there are no observations of binary white dwarf systems with near Chandrasekhar combined mass. In the DD scenario one would expect that the peak of the white dwarf distribution at 0.6 would lead to a secondary peak at 1.2 due to a coalescence of binary white dwarfs. Indeed in figure. 2 there seems to be such a secondary peak but it is known [19] that this is purely an artifact of the treatment of the high mass tail.

Significant discussions with Peter Biermann, Ken Olum, and Akos Bogdan contributing to the work of this paper are gratefully acknowledged.

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