Gravitational collapse of a susy star

L. Clavelli*

Department of Physics and Astronomy

University of Alabama

Tuscaloosa AL 35487

February 1, 2006

Abstract

The evidence for a positive vacuum energy in our universe suggests that we might be living in a false vacuum destined to ultimately decay to a true vacuum free of dark energy. At present the simplest example of such a universe is one that is exactly supersymmetric (susy). It is expected that the nucleation rate of critically sized susy bubbles will be enhanced in regions of high density such as in degenerate stars. The consequent release of energy stored in Pauli towers provides a possible model for gamma ray bursts. Whether or not all or any of the currently observed bursts are due to this mechanism, it is important to define the signatures of this susy phase transition. After such a burst, due to the lifting of degeneracy pressure, the star would be expected to collapse into a black hole even though its mass is below the Chandrasekhar limit. Previous studies have treated the star as fully releasing its stored energy before the collapse. In this article we make an initial investigation of the effects of the collapse during the gamma ray emission.

PACS numbers:12.60.Jv, 95.85.Pw

---

*lclavell@bama.ua.edu

1 Introduction

Present indications are that we live in a broken-susy universe with a positive vacuum energy density

$$\epsilon = 3560 MeV/m^3 \tag{1.1}$$

leading to an acceleration in the expansion of the universe. This has given new fuel to the idea that we live in a false vacuum and that the universe will ultimately make a transition to a lower energy vacuum. If, as suggested by string theory, there is an exactly supersymmetric, zero vacuum energy universe that is dynamically connected to the broken susy universe, the final phase of the universe could be exactly supersymmetric. The theory of vacuum decay was pioneered some decades ago by Coleman [1]. In this theory, bubbles of true vacuum are continually being nucleated in the false vacuum. Most of these are quite small and are immediately quenched. However, when one appears with radius greater than some critical radius

$$R_c = \frac{3S}{\epsilon} \tag{1.2}$$

it will rapidly grow to take over the universe. Here, S is the surface tension of the bubble assumed to be independent of bubble size. The probability per unit time per unit volume to produce a bubble of radius R_c or greater and, therefore, to effect a phase transition to the true vacuum is given [1] in the form

$$\frac{d^2P}{dt d^3r} = Ae^{-B(vac)} \tag{1.3}$$

where, assuming a thin wall between the phases,

$$B(vac) = \frac{27\pi^2 S^4}{2\epsilon^3} \quad . \tag{1.4}$$

A first look at the environment of a susy universe has been reported in [2] but, for sufficiently large S and small A, the transition is not likely to take place in the near future. In much of this article we use the solar mass, $M_\circ = 1.2 \cdot 10^{60}$ MeV, and earth radius, $R_E = 6.38 \cdot 10^6$ m as convenient units. Factors of $\hbar$ and c are sometimes left implicit.

Reasonable arguments [3, 4] have been given that the transition rate could be enhanced in dense matter. This enhancement can be implemented in a natural way [5] if the above equations are modified in dense matter by replacing ϵ by the total energy advantage per unit volume of trading the broken susy phase for the exact susy phase, i.e.

$$\epsilon \rightarrow \epsilon + \Delta\rho \tag{1.5}$$
$$R_c \rightarrow \frac{3S}{\epsilon + \Delta\rho}$$

where $\Delta\rho$ is the difference in the ground state matter densities between the broken susy phase and the exact susy phase as shown in fig. 1.
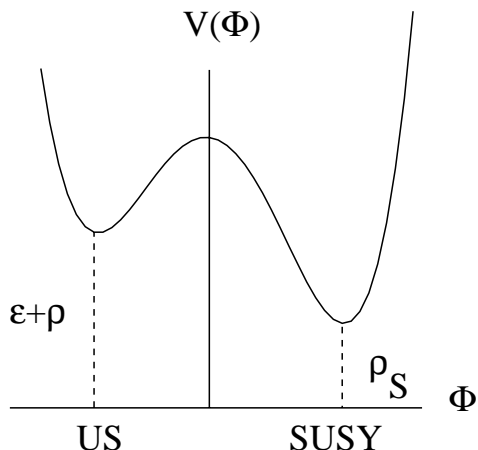
Figure 1: The effective potential showing the false vacuum of broken susy and the true vacuum of exact susy.

The difference $\Delta\rho$ is the fermionic excitation energy density. The parameter controlling the exponential factor in the transition rate would then be

$$B = \frac{27\pi^2 S^4}{2(\epsilon + \Delta\rho)^3} \quad .$$ (1.6)

The value of $\Delta\rho$ in a white dwarf star is calculated as follows. In a degenerate electron gas of N_e electrons in a volume V the Fermi momentum is

$$p_F = \left(\frac{3\pi^2 N_e}{V}\right)^{1/3}\hbar$$ (1.7)

with, assuming equal numbers of neutrons and protons,

$$N_e/V = \frac{\rho}{2M_N}f_e$$ (1.8)

Here M_N is the nucleon mass and f_e is the electron to proton ratio. Before the phase transition f_e is equal to unity but afterwards it decreases as electron pairs convert to selectron pairs. At any stage in the conversion process,

$$e^- e^- \rightarrow \tilde{e}^- \tilde{e}^- \quad ,$$ (1.9)

one would have, by charge conservation, $N_P = N_e + N_{\tilde{e}}$, so

$$f_e = \frac{N_e}{N_e + N_{\tilde{e}}}$$ (1.10)

In previous studies [6], this conversion process reducing f_e to essentially zero was assumed for simplicity to take place at fixed stellar radius. In actuality, of course, as the Fermi degeneracy

3

is lifted the star will tend to shrink under the force of gravity. Without attempting at this time to treat the full time-dependent problem we give in this paper some attention to sequential stages in the gravitational collapse of a susy star.

The average electron kinetic energy in the degenerate gas of a white dwarf star is

$$< E > -m = m\left(-1 + \frac{3}{8}\sqrt{(1+z)}(2+1/z) - \frac{3}{8z^{3/2}}\ln\frac{\sqrt{z}-1+\sqrt{1+z}}{\sqrt{z}+1-\sqrt{1+z}}\right) \quad . \quad (1.11)$$

where

$$z = \frac{p_F^2}{m^2} \quad . \tag{1.12}$$

In the limit of high Fermi momentum relative to the electron mass, this is

$$< E > -m = \frac{3p_F}{4} \quad . \tag{1.13}$$

while for Fermi momentum low compared to the electron mass,

$$< E > -m = \frac{3}{10}mz \quad . \tag{1.14}$$

The kinetic energy density, which is equal to the difference in ground state energy densities between the broken susy state and the exact susy state, is then

$$\Delta\rho(r) = (< E > -m)\frac{\rho}{2M_N}f_e \tag{1.15}$$

We neglect for now contributions from nuclear excitation energies. In [6], we used for simplicity the high Fermi momentum limit 1.13. In our current work the exact expression 1.11 is used.

In the broken susy phase ($f_e = 1$), a white dwarf star is supported by a balance between the outward electron degeneracy pressure gradient and the inward gravitational pressure gradient. The mass and radius of a white dwarf star are then each determined by the density at the center leading to a unique mass-radius relation. These relationships, originally calculated by Chandrasekhar [7] are shown in [6] at zero temperature. In section II we calculate the corresponding relations for several cases of f_e below unity but assumed uniform over the volume of the star. In the case of uniform f_e, the star would collapse at constant mass though these stages of decreasing f_e (assuming the radiation is a negligible fraction of its rest energy during the process). As the radius of the star decreases, the central density and, therefore, the energy stored in the Pauli tower of electrons increases even though the fraction of Fermionic electrons is decreasing. Thus the collapse provides a natural pumping mechanism from gravitational energy into the stored Pauli energy. It is found that the central density increases so rapidly during this process that a central core of the star decouples and becomes a black hole while f_e is still relatively high.

This calculation, however, is obviously oversimplified since the critical radius of the susy bubble is expected to be small compared to the radius of the star. f_e would drop rapidly to

near zero within the bubble which would then only slowly expand to engulf the star. It also neglects the radiation pressure due to the Pauli energy released within the bubble.

In section III we slightly refine the toy model to take into account in an adiabatic and very approximate way a non-uniformity of f_e and the effect of nuclear energy release. We assume that the susy bubble begins at the center of the star with a critical radius and then proceeds through stages of greater radius. Inside the bubble the degeneracy pressure is set to zero but there is a balance between the gravitational pressure gradient and the radiation pressure gradient due to the nuclear energy release. Outside the bubble wall there is the usual balance between the degeneracy pressure gradient and the gravitational pressure gradient. Photons of energy less than the Fermi energy of the electron gas are assumed to pass freely out of the star due to the Landau-Pomeranchuk effect [8] but most of the radiation is trapped in the regions of high density and only slowly cools. We neglect this cooling within the bubble which, of course, would ultimately lead again to a black hole collapse due to the absence of Fermion degeneracy. The sudden dumping of nuclear energy near the stellar center might also set up density waves throughout the star which could affect the growth of the bubble. The complete time dependence of this process would require a much more elaborate Monte-Carlo calculation than we can attempt at present.

In both of the simplified models treated here, the energy released in the collapse could be significantly greater than that calculated in [6] although this depends to some extent on how much of the energy release is trapped within the developing black hole. In both cases the mass of the resulting black hole would still be significantly below the Chandrasekhar limit which was one of the prime predictions of the original susy star model.

Section IV is devoted to a summary of our current level of understanding of stellar behavior following a phase transition to exact supersymmetry.

2   Stages of uniformly decreasing N_e/N_p

Following a susy phase transition, the degeneracy pressure in a dense star will decrease as electron pairs convert to scalar electron (selectron) pairs via photino exchange. In this section we neglect radiation pressure and treat, as a toy model, the case of a uniform ratio of electron to selectron numbers throughout the star. Such a model might be more realistic if the surface tension of eq. 1.6 was such as to make the critical radius comparable to the radius of the dense star.

These approximations will be somewhat relaxed in the subsequent section. At present we seek to determine the density profile of the star as a function of f_e at fixed stellar mass. We follow the Chandrasekhar calculation but allow for a sequence of decreasing f_e ratios.

The gravitational pressure gradient is

$$\frac{dP_G}{dr} = -\rho(r)G\frac{M(r)}{r^2} \tag{2.1}$$

where Newton's constant, in convenient units of earth radius and solar mass, is

$$G = 2.34 \cdot 10^{-4} R_E c^2 M_0^{-1} \quad . \tag{2.2}$$

At zero temperature, a stable density profile is defined by a balance between this gradient and the gradient of degeneracy presssure. The degeneracy pressure is

$$P_d = af(x) \tag{2.3}$$

where

$$x = \frac{P_F}{mc} = b(\rho f_e)^{1/3} \tag{2.4}$$

and

$$a = \frac{m^4 c^5}{3\pi^2 \hbar^3} = 0.0165 R_E^{-3} M_0 c^2 \tag{2.5}$$

$$b = \frac{2\pi\hbar}{mc} \left(\frac{3}{8\pi m_N \mu_e}\right)^{1/3} = 1.6 R_E M_0^{-1/3} \quad .$$

Here $\mu_e = A/Z$ which we take equal to 2 as in a Carbon or Oxygen star. These formulae are straightforward generalizations to f_e below unity from standard presentations as, for example, given in [9]. The function $f(x)$ is given by

$$f(x) = \frac{1}{8} \left(x(2x^2 - 3)\sqrt{x^2 + 1} + 3\sinh^{-1}(x) \right) \tag{2.6}$$

so that

$$\frac{dP_d}{dr} = \frac{ab}{3} (f_e \rho)^{-2/3} f'(x) \frac{d(\rho f_e)}{dr} \quad . \tag{2.7}$$

where

$$f'(x) = \frac{x^4}{\sqrt{1 + x^2}} \quad . \tag{2.8}$$

The balance of gravitational and degeneracy pressure requires that

$$f_e \frac{dw}{dr} = -\frac{G}{ab^3} \frac{M(r)}{r^2} \tag{2.9}$$

where

$$w = \sqrt{1 + b^2(\rho f_e)^{2/3}} \quad . \tag{2.10}$$

Since

$$M(r) = 4\pi \int_0^r r'^2 dr' \rho(r') \tag{2.11}$$

The equilibrium density profile satisfies the second order differential equation

$$\frac{1}{r^2} \frac{d}{dr} f_e r^2 \frac{dw}{dr} = -\frac{4\pi G}{ab^3} \rho(r) \quad . \tag{2.12}$$

We find it, however, more convenient to use eq.2.9. We begin by specifying some central density $\rho(0)$ with $M(0) = 0$. We then integrate out in steps of $dr = M_0 \cdot 10^{-5}$ putting

$$M(r + dr) = M(r) + 4\pi\rho(r)r^2 dr \qquad (2.13)$$

and

$$w(r + dr) = w(r) + \frac{dw}{dr}dr \quad . \qquad (2.14)$$

The gravitational energy is zero at the center and is incremented according to

$$U(f_e, r + dr) = U(f_e, r) - 4\pi G r dr M(r)\rho(r) \quad . \qquad (2.15)$$

The process terminates at the point at which the density drops to zero and this defines the radius and mass of the star as well as its total gravitational energy.

We consider a star with central density $\rho_0 = 8.3 M_0 R_E^{-3}$. Before the susy phase transition ($f_e = 1$) such a star will be stable at a mass of $M = 1.09 M_0$ and a radius of $R(f_e = 1) = 0.73 R_E$. Its total gravitational potential energy is found to be

$$U(1, R) = -1.6 \cdot 10^{55} ergs \quad . \qquad (2.16)$$

We then decrease f_e by a small amount and repeat the process increasing $\rho(0)$ so that the total mass remains the same. These stages of stepwise decreasing f_e are characterized by a sequence of decreasing radii $R(f_e)$, increasing central densities $\rho_0(f_e)$, and increasingly negative gravitational energy. Due to the increasing density, the local Fermi momenta as given by eqs.1.7 and 1.8 increase in this process even though the electron fraction f_e is decreasing. Thus the collapse provides a natural pumping mechanism from gravitational energy into the energy of the electron cloud. This energy is released as electrons convert to selectrons which are not bound by the Pauli principle. The resulting photons of energy less than the Fermi energy escape from the star with little absorption due to the Landau-Pomeranchuk effect. This mechanism for a fast escape of photons from a star was first pointed out by Takahashi et al. [10].

The Schwarzschild radius for a spherical object of mass $M(r)$ is

$$R_S(r) = \frac{2GM(r)}{c^2} \quad . \qquad (2.17)$$

If at any point in the integration,

$$r < R_S(r) \qquad (2.18)$$

the stellar core decouples and collapses to a black hole. This core collapse is analogous to that of the collapsar model [11] of long gamma ray bursts.

The collapsar model is, however, most obviously applicable to heavy stars whereas we are dealing with a near solar mass star. Attempts to describe short bursts ($\tau < 2$ s) within the standard model are often based on the hypothesis that an incipient black hole produced
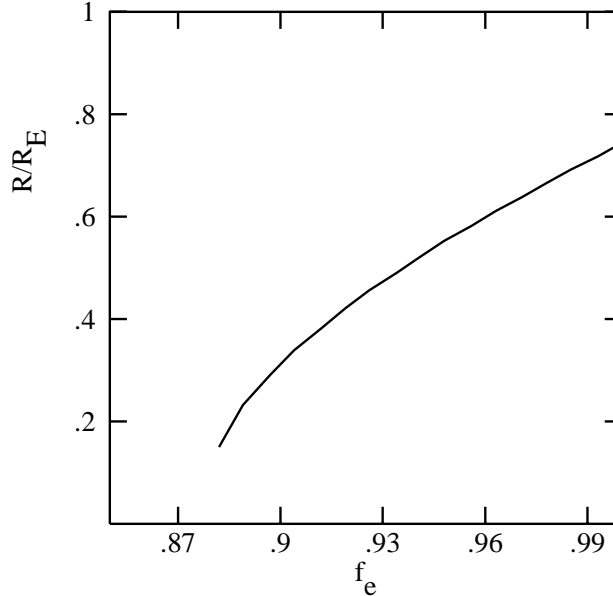
Figure 2: The stellar radius in units of R_E as a function of electron fraction in the toy model of section II.

by rapid accretion onto a neutron star could emit jets of the requisite mean photon energy, total energy, and collimation. In the susy phase transition model it is possible even for an isolated star to collapse and one avoids the possible problems of how to rapidly mix the accreted material to kindle fusion and of how to rapidly extract sufficient energy at the proper wavelengths.

For our chosen stellar mass of $1.09 M_0$ we find that core collapse happens when f_e drops below 0.88 at a radius of $2.7 \cdot 10^{-4} R_E$. The mass at that radius is roughly $0.6 M_0$, well below the Chandrasekhar limit, $(1.4 M_0)$, below which no black holes would be expected in the standard model.

In figure 2, we show the radius of the stable configuration of fixed mass as the electron ratio falls below unity down to 0.88. One sees that, at this point of core collapse, there is still a significant electron degeneracy.

Figure 3 plots density in units of M_0/R_E^3 versus radius in units of R_E for five different values of f_e.

Figure 4 shows as a function of f_e the total kinetic energy of the degenerate electron gas in units of 10^{51} ergs (solid curve), the mean electron kinetic energy in MeV (dashed curve), and the negative of the gravitational energy in units of 10^{55} ergs (dot-dashed curve). Each point on the curves represents a zero temperature equilibrium stage in which the degeneracy pressure gradient balances the gravitational pressure gradient. The release of gravitational energy as f_e decreases is more than sufficient to replenish the energy of the Fermi sea. From
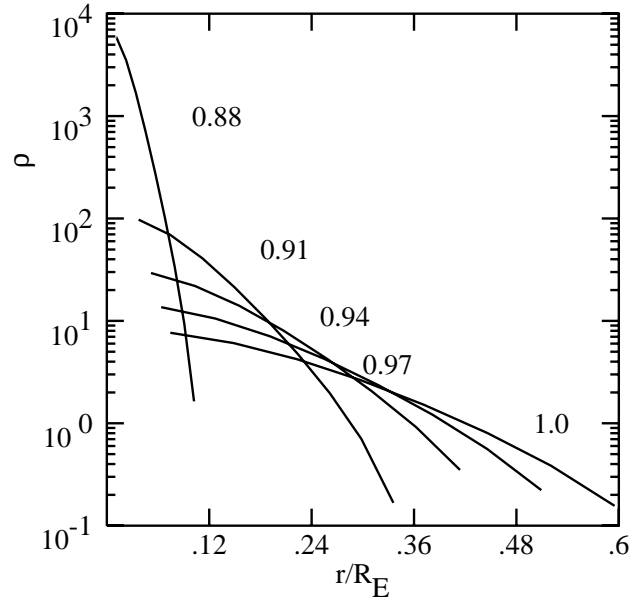
8

Figure 3: The stellar density profile in units of $M_0 R_E^{-3}$ as a function of radius for five different indicated values of electron fraction f_e. The sharp increase in the central density as f_e approaches 0.88 is apparent.
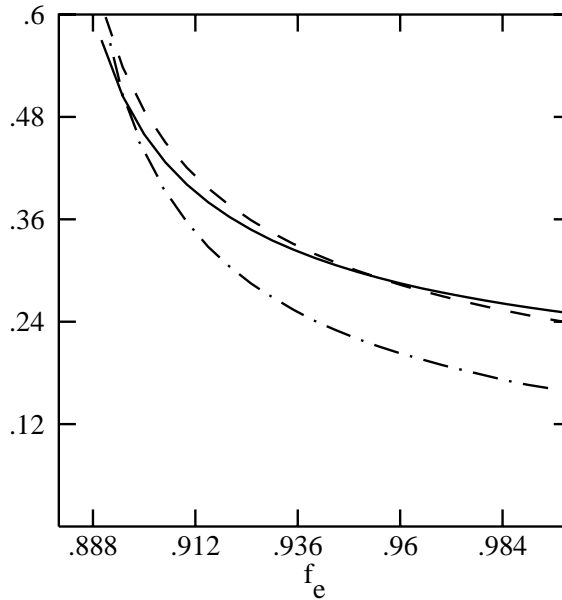

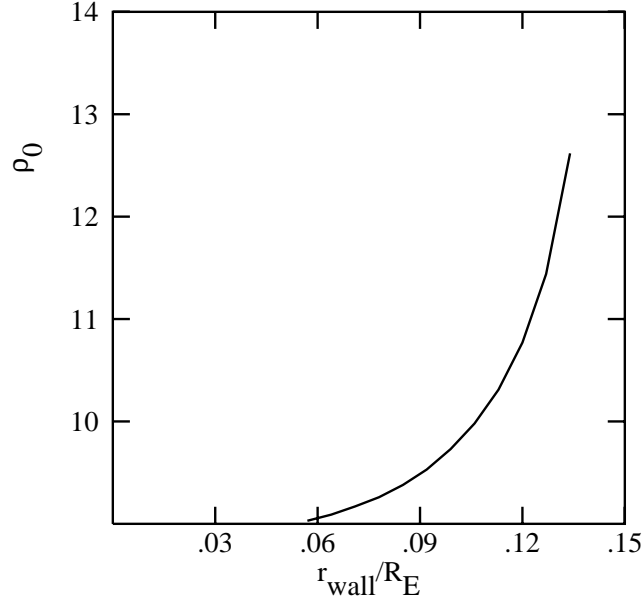Figure 4: various stellar properties as a function of electron fraction. See text.

Figure 5: The central density in units of $M_0 R_E^{-3}$ as a function of bubble wall radius requiring that the total mass is fixed at $1.09 M_0$.

the dot-dashed curve of figure 4 one can see that about 10^{54} ergs of gravitational energy is released before the point of instability is reached. How much of this escapes and how much is swallowed by the incipient black hole is dependent on the temporal behavior of the collapse which is left to a later analysis. However, it is clear that the total radiation is potentially more energetic than the estimates of [5],[6]. The photons of energy below the Fermi energy should freely escape from the star. We can see from the dashed curve of figure 4 that the average energy of these photons will typically lie in the 0.1 MeV to 1 MeV as in the observed gamma ray bursts. In this section we have neglected the additional energy release from fusion induced by the gravitational energy dumping.

3   Growth of the susy phase from a small bubble

In this section we will assume the phase transition takes place in a small bubble in which the f_e ratio drops immediately to approximately zero. The energy release from the Pauli towers in a Carbon or Oxygen nucleus is estimated to be about 0.5 MeV per nucleon.

$$\varepsilon = \frac{0.5 MeV}{m_N c^2} \approx 5 \cdot 10^{-4} \quad . \tag{3.1}$$

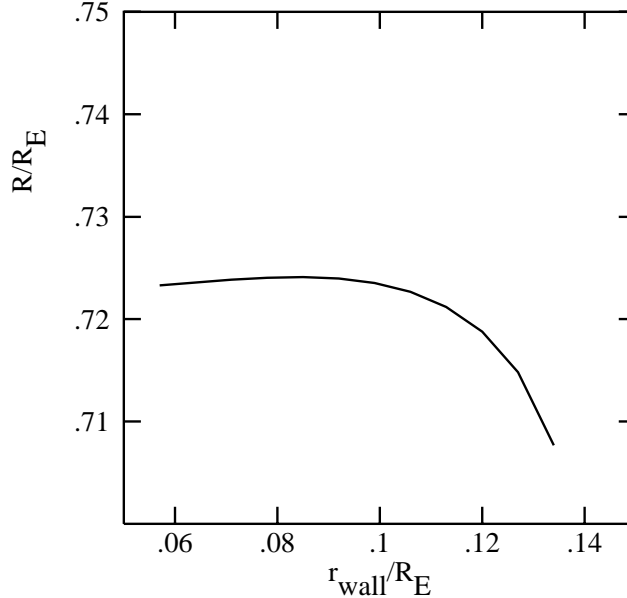The radiation pressure is 1/3 of the radiative energy density so the radiative pressure

10

Figure 6: The total stellar radius as a function of wall radius both given in units of earth radius.

gradient is then

$$\frac{P_{rad}}{dr} = \frac{\varepsilon c^2}{3} \frac{d\rho}{dr} \tag{3.2}$$

Within the susy bubble this radiative pressure gradient must balance the gravitational pressure gradient eq.2.1. In a time dependent treatment the radiative pressure will gradually decrease and there will be a gravitational collapse. In this article we do not attempt to treat this cooling but leave the time dependent processes to future study.

Outside of the bubble we will have $f_e = 1$ and there can be a balance between the degeneracy and gravitational pressure gradients. We consider a sequence of increasing wall radius with $f_e = 0$ for $r < r_{wall}$ and $f_e = 1$ for $r > r_{wall}$.

In figure 5 we show the central density as a function of wall radius for a stellar mass of 1.09 M_0. As the bubble grows the central density peaks and, for this mass, the system becomes unstable at a wall radius of 0.14 R_E at which no equilibrium configuration is found.

As the susy bubble grows the total radius of the star in this model varies only slowly at first as shown in figure 6. However, as one approaches the instability the radius drops sharply. As discussed in [5] and [6], the bubble is confined within the star and does not grow to engulf the universe.

As would be expected in the model where the electron fraction f_e is discontinuous at a sharp wall boundary, the equilibrium density distribution shows a break at the boundary.
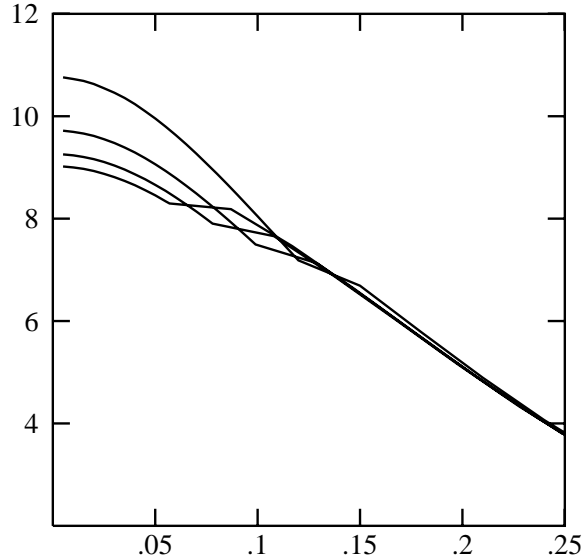
11

Figure 7: The density distribution in units of $M_0 R^{-3}$ as a function of radius for four values of the wall radius (see text). The slope of the density distribution is discontinuous at the bubble wall.

In figure 7 we show the variation of density with radius for four values of r_{wall}, namely r_{wall} equal to $0.057 R_E$, $0.078 R_E$, $0.099 R_E$, and $0.120 R_E$. The central density increases with wall radius.

4 Summary

The analysis of the current paper presents several new aspects of the behavior of a dense star following a phase transition to exact susy. The main results are the curves showing how the radius varies with susy fraction as fermions convert in pairs to scalars and the observation of an instability while there is still a large amount of Fermi degeneracy remaining.

Although we have not analysed the time dependence of the problem, the picture emerging from the present study is as follows. The conversion of Fermions to Bosons tends to lift the degeneracy pressure which supports dense stars in the broken-susy world. This tends to decrease the stellar radius and disproportionately increase the the central density which in turn tends to maintain or increase the mean and maximum energy of the Fermi gas. The additional energy is provided by the shift in gravitational potential energy toward more negative values. The gravitional energy released goes first into replenishing the Fermi sea. Additional photons with energy below the Fermi energy pass freely out of the star while more energetic photons increase the temperature but are trapped in regions of high density

and only slowly escape. This trapping of photons in dense matter is a familiar property of non-degenerate matter such as stellar interiors at elevated temperatures. The consequent radiation pressure slows the gravitational collapse and increases the amount of energy that can potentially escape before the star is engulfed in a black hole.

The Bose enhancement of the selectron final state of the conversion process has been shown [12] to lead to some amount of jet structure but, without replenishment of the Fermi sea, this would not be as efficient as necessary if the bursts are totally jet-like as suggested by some. The release of gravitational energy serves as a natural pumping mechanism to provide this replenishment. Whether or not the effectiveness of this pumping mechanism will survive the future incorporation of dynamical effects into the model is, however, uncertain. On the other hand much additional energy could be released which may not be totally jet-like but might be part of a quasi-isotropic burst much more energetic than implied by a purely jet like emission. The two simple quasi-adiabatic models presented here suggest that the Fermi sea is only partially depleted before the star becomes unstable and collapses. This supports the idea that the Landau-Pomeranchuk effect plays an important role in the rapid gamma ray extraction. The mean photon energy expected in this model is in the range of currently observed gamma ray bursts and the total energy in the Fermi sea is roughly the observed burst energy assuming a strong collimation. The possibility of continually replenishing the Fermi sea as the susy conversion proceeds allows for the possibility of emitting more energy than could be obtained by a single emptying of the Fermi sea. However, since the emitted energy derives from the susy bubble which is still a small fraction of the star prior to the instability, whether or not the total energy emitted is greater than the earlier estimates of [5, 6] could be sensitive to the time dependent details of the collapse which are not treated in the present quasi-adiabatic approximation.

The main remaining problem left for future study in the susy star model is a complete monte-carlo of the time dependence of the coupled bubble growth and stellar collapse following the phase transition. This should allow a more complete prediction of the total energy released and resolve the question of jet structure in this model.

References

[1] S. Coleman, *Phys. Rev. D15*, 2929 (1977);C.G. Callan and S. Coleman, *Phys. Rev. D16*, 1762 (1977); S. Coleman and Frank DeLuccia, *Phys. Rev.* D21, 3305 (1980).

[2] L. Clavelli, hep-th/0508207.

[3] A.S. Gorsky and V.G. Kiselev, *Phys. Lett. B304*, 214 (1999).

[4] M.B. Voloshin, *Phys. Rev. D49*, 2014 (1994).

[5] L. Clavelli and G. Karatheodoris, hep-th/0403227, *Phys. Rev. D72*, 035001 (2005).

[6] L. Clavelli, hep-ph/0506215, *Phys. Rev.D72* ,055005 (2005).

[7] S. Chandrasekhar, *Introduction to the Study of Stellar Structure*, University of Chicago Press, (1939).

[8] L. D. Landau and I.J. Pomeranchuk, Dokl. Akad. Nauk SSSR *92*, 535 (1953); also in Collected Papers of L.D. Landau, edited by D. Terhaar, Gordon and Breach, NY, p586 (1965).

[9] M. Harwit, *Astrophysical Concepts*, Wiley and Sons, New York, (1973).

[10] Y. Takahashi, L.W. Hillman, and T. Tajima, Relativistic Lasers and High Energy Astrophysics in High Field Science, Plenum Press, NY (1999).

[11] S.E. Woosley, A.I. Macfadyen, and A. Heger, astro-ph/9909034.

[12] L. Clavelli and I. Perevalova, Phys. Rev. D71, 055001 (2005).