Extended Higgs Models and a Transition to Exact Susy

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Abstract. String landscape ideas and the observation of a positive vacuum energy in the current universe suggest that there could be a future transition to an exactly supersymmetric world. Atomic and molecular binding in this susy background probably require that electroweak symmetry breaking survives the transition. Among several susy higgs models that have been discussed, one stands out in this regard. Thus, the higgs structure that is revealed at the LHC could have strong consequences for the type of bulk matter that may arise in a future susy universe.

The observations of a small but positive vacuum energy in our universe plus the strong indications that, in its early moments, the universe made transitions from states of much higher vacuum energy, raise the question of whether there are further transitions to be expected within our horizon. If so, it is natural to ask what properties this future universe might have.

We seem to be living in a bubble that formed some 13.7 billion years ago and that, after passing through many meta-stable states in a brief inflationary era, transitioned to our current calm umiverse which is, nonetheless, still inflating with a vacuum energy density measured to be

$$\epsilon_{now} = 3.560 GeV/m^3 = (.0023 eV)^4 \quad . \qquad (1)$$

This is some 124 orders of magnitude less than the natural value, M^4_{Planck} that might have been expected for this quantity but it is known [1] that arriving in such a calm universe was a prerequisite for the evolution of advanced life forms. From a physics point of view it is, however, necessary to ask what circumstances might have made this early history of our universe not extremely improbable. This seems to lead inevitably to speculation about possible regions of the universe outside of causal contact with us.

For example, the scenario of eternal inflation [2] proposes that the universe is infinite in spatial and temporal extent and that, consequently, however low the probability of life is per unit of space-time volume, there are infinite numbers of civilizations in space-time that are similar to ours. This picture requires that there is an equilibrium established in the "multiverse" and that the probability of "jumping up" to a state of higher vacuum energy is in statistical balance with that of "jumping down". It is also thought by many in this school that there are infinitely many possible states of the universe that are massively anti-DeSitter, i.e. possessing enormously negative vacuum energy density ϵ. Such states would collapse in a big crunch on a time scale of $1/\sqrt{24\pi G_N|\epsilon|}$. The probability of a transition to such a deeply negative vacuum energy should also be quite high raising the question of why our current universe has persisted for its multi-billion year lifetime. Furthermore, naive physical intuition suggests that transitioning to a state of lower energy density should be vastly more probable than jumping up and it would seem that additional theoretical analysis would need to be done in this picture to establish a result preventing the universe everywhere from evolving inevitably to the lowest possible vacuum energy. In any case, in the absence of experimental confirmation, one is free to ask whether other possibilities exist.

We study an alternate scenario in which the universe has a supersymmetric (susy) gound state of exactly zero vacuum energy. Examples of such universes are provided by the five original superstring theories but we prefer to think in terms of a simple supersymmetric extension of the standard model. In this model the visible universe should eventually make a transition to the susy ground state. One could envision an inhomogeneous universe where, outside of our horizon, shells of higher vacuum energy from the inflationary era are inflating rapidly but without sufficient matter density to spawn galaxies. Note that in the standard false vacuum decay theory [3], vacuum energy goes into the bubble wall and not into creating matter. Outgoing shells may be unlikely to collide sufficiently to create significant matter.

Some attention has been given to the possible properties of a future susy universe [4,5]. The primary fea-

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ture of such a universe is a weakening of the Pauli Principle due to the degeneracy of fermions and bosons. Susy atoms, if they exist, would have entirely s-wave ground states. As in our universe, quantum mechanics predicts that all binding energies are proportional to the electron (or common electron/selectron) mass. The mean radii of susy atoms would be inversely proportional to this mass. Thus, unless electroweak symmetry breaking (EWSB) survives the transition to the future susy universe providing masses, no electromagnetic bound states could be expected.

The time scale [4] for the transition to take place is governed by the behavior of the cube of the cosmological scale factor

$$\tau = \sqrt{\frac{1}{24\pi G_N \epsilon}} = 5.61 \cdot 10^9 yrs \quad . \tag{2}$$

We have investigated several popular phenomenological susy models from this point of view [6]. One could start from the most general renormalizable superpotential, W, involving the mimimal pair of higgs doublets plus a singlet field, S. To avoid the uncontrolled growth of MSSM related acronyms We refer to this most general model with a single extra singlet as the Singlet Extended Susy Higgs Model (SESHM).

$$W = \lambda \left(S(H_u \cdot H_d - v^2) + \frac{\lambda'}{3} S^3 + \frac{\mu_0}{2} S^2 \right) \quad . \tag{3}$$

This leads to a scalar higgs potential with F terms of the form

$$V_F = \lambda^2 \left(|H_u \cdot H_d - v^2 + \lambda' S^2 \right.$$
$$\left. + \mu_0 S|^2 + |S|^2 (|H_u|^2 + |H_d|^2) \right) \tag{4}$$

and to D terms of the form

$$V_D = \frac{g_1^2 + g_2^2}{8} \left(|H_d|^2 - |H_u|^2 \right)^2$$
$$+ \frac{g_2^2}{2} \left(|H_d|^2 |H_u|^2 - |H_u \cdot H_d|^2 \right) \tag{5}$$

where g_1 and g_2 are the $U(1)$ and $SU(2)$ gauge couplings.

There could also be susy breaking "soft" higgs mass terms and other soft terms proportional to terms in the superpotential. These soft terms are something of an embarrassment for susy theory since their origin is not understood. The problem is often swept under the rug by referring to some unknown susy breaking mechanism in a "hidden sector" which is communicated gravitationally or otherwise to our sector. However, interactions in string theory hidden sectors are similar to those in our sector and, if we understood their origin in a hidden sector, the same mechanism should be available in our sector. No treatment of susy breaking which appeals to soft susy breaking terms can be complete unless the origin of these terms is also understood.

We would prefer to assume, as we tentatively do here, that these terms are absent unless forced on us for phenomenolgical reasons. For example, in the Minimal Susy Standard Model (MSSM) they are necessary to obtain consistency with experimental constraints. In any case, the soft terms will disappear if susy breaking vanishes in both sectors though they are relevant in the broken susy phase.

The SESHM was first written some thirty years ago [7]. It was noted then that, unless the soft terms were somehow essential, the susy breaking phase was at best meta-stable with the true minimum being exactly supersymmetric. Since then various sub-spaces of the λ', μ_0 parameter space have been investigated under various names; see for example [8,9].

Some of these limiting cases and the corresponding labels are

v, $\mu_0 \to 0$: $W \to NMSSM$, The "Next to Minimal Susy Standard Model",

λ', $\mu_0 \to 0$: $W \to nMSSM$, the "nearly Minimal Susy Standard Model"

λ', μ_0, $v \to 0$: $W \to UMSSM$, a $U(1)$ extended MSSM

It seems that the region of non-zero μ_0 has not as yet been subjected to phenomenological scrutiny, perhaps because this parameter can be consistently, though with some loss of generality, set to zero. Because of the symmetry between H_u and H_d, the D terms vanish at the minima. At our present level of investigation we have ignored phases, thus postponing the possibility of addressing CP violation.

The extrema of the potential define vacuum expectation values $v_0 = <H_u> = <H_d>$ and $S_0 = <S>$ satisfying

$$\frac{1}{\lambda^2} \frac{\partial V_F}{\partial S}\Big|_0 = 0$$
$$= (2\lambda' S_0 + \mu_0)(v_0^2 - v^2 + \mu_0 S_0 + \lambda' S_0^2) + 2v_0^2 S_0 \tag{6}$$

$$\frac{1}{\lambda^2} \frac{\partial V_F}{\partial H_u}\Big|_0 = 0$$
$$= v_0(v_0^2 - v^2 + \mu_0 S_0 + (\lambda' + 1)S_0^2) \quad . \tag{7}$$

The solutions are

Solution 1: $v_0 = v$, $S_0 = 0$
This is one of the two grounds state of the model and corresponds to an exact supersymmetry (vanishing vacuum energy) with EWSB ($v_0 \neq 0$).

Solution 2: $v_0 = 0$, $S_0 = \frac{-\mu_0 \pm \sqrt{\mu_0^2 + 4\lambda' v^2}}{2\lambda'}$
This solution is also supersymmetric with a vanishing vacuum energy at the minimum but with no EWSB.

Solution 3: $v_0 = 0$, $S_0 = \frac{-\mu_0}{2\lambda'}$
This corresponds to a broken susy with no EWSB.

Solution 4: A solution to the minimization equations with non-zero v_0 and S_0. This corresponds to the phase most close to our current universe with broken susy and EWSB. By eliminating $\mu_0 S_0$ from eqs. 6 and 7 we find the conditions

$$S_0^2 = \frac{3v_0^2 - v^2}{\lambda' - 1} \tag{8}$$

and

$$\mu_0^2 = \frac{4(\lambda' v^2 - v_0^2(2\lambda' + 1))^2}{(\lambda' - 1)(3v_0^2 - v^2)} \quad . \qquad (9)$$

These two equations determine v_0 and S_0 in terms of λ' and μ_0^2. One can see that eqs. 8 and 9 each predict $v_0^2 > v^2/3$ for $\lambda' > 1$ and $v_0^2 < v^2/3$ for $\lambda' < 1$.

For these extrema to be true minima, one would have to require that the parameters are such that the eigenvalues of the matrix of the second derivatives of the potential are all positive at the solutions of eqs. 6 and 7. Since the potential is positive definite or zero, any localized solution with vanishing vacuum energy, $V_F(0)$, is necessarily a minimum. For the broken susy case, we must study the mass squared matrix

$$M^2 = \begin{vmatrix} \alpha + \zeta & \gamma - \zeta & \delta \\ \gamma - \zeta & \alpha + \zeta & \delta \\ \delta & \delta & \beta \end{vmatrix}$$

where

$$\alpha = \left.\frac{\partial^2 V_F}{\partial H_u^2}\right|_0 = 2\lambda^2(v_0^2 + S_0^2) \qquad (10)$$

$$\gamma = \left.\frac{\partial^2 V_F}{\partial H_u \partial H_d}\right|_0 = 2\lambda^2(v_0^2 - S_0^2) \qquad (11)$$

$$\delta = \left.\frac{\partial^2 V_F}{\partial H_u \partial S}\right|_0 = 2\lambda^2 v_0 \left(2(\lambda' + 1)S_0 + \mu_0\right) \quad (12)$$

$$\begin{aligned} \beta = \left.\frac{\partial^2 V_F}{\partial S^2}\right|_0 = 2\lambda^2 \big(&-2\lambda' S_0^2 \\ &+ (2\lambda' S_0 + \mu_0)^2 + 2v_0^2\big) \end{aligned} \qquad (13)$$

$$\zeta = \left.\frac{\partial^2 V_D}{\partial H_u^2}\right|_0 = (g_1^2 + g_2^2)v_0^2 \quad . \qquad (14)$$

The g_i are the electroweak coupling constants and ζ is proportional to the Z mass squared. The eigenvalues of the mass squared matrix satisfy

$$m_1^2 = 4\lambda^2 S_0^2 + 2\zeta \quad , \qquad (15)$$

$$m_2^2 + m_3^2 = \alpha + \beta + \gamma \quad , \qquad (16)$$

and

$$m_2^2 m_3^2 = \beta(\alpha + \gamma) - 2\delta^2 \quad . \qquad (17)$$

The first squared mass is positive definite and corresponds to the eigenvector

$$\Psi_1 = \frac{H_u - H_d}{\sqrt{2}} \quad . \qquad (18)$$

The positivity of m_2^2 and m_3^2 puts constraints on the parameter space of λ' and μ_0, namely

$$m_2^2 + m_3^2 = 4\lambda^2 v_0^2 + \beta > 0 \qquad (19)$$

and

$$\begin{aligned} m_2^2 m_3^2 = \frac{16\lambda^4 v_0^2}{\lambda' - 1} &\left(-v_0^2(6\lambda' + 3) + v^2(\lambda' + 2)\right) \\ &> 0 \quad . \end{aligned} \qquad (20)$$

A simultaneous solution of eqs. 19 and 20 requires that $\lambda' < -2$ and that

$$v_0^2 < v^2 \frac{\lambda' + 2}{6\lambda' + 3} \quad . \qquad (21)$$

For example, it is clear from eq. 13 that negative λ' guarantees the positivity of eq. 19. It requires some analysis to see that no solutions with positive λ' exist except, perhaps, over a set of measure zero. Thus the broken susy phase with EWSB lies between the two exact susy phases. Assuming equality of the Yukawa couplings in the various phases, the common electron and selectron mass in the exact susy phase with EWSB would be v/v_0 times greater than in the broken susy phase. Thus the transition would be endothermic requiring an extra input of energy. For elements above helium, this energy could come from the energy released from the Pauli towers as nucleons convert to scalar nucleons.

An interesting result, emphasized in [6], is that the NMSSM and the UMSSM, like the MSSM, have no EWSB in the susy limit (i.e. $v_0 = 0$ in the susy phase). Only models with non-zero v offer possibilities for atomic and molecular structures in a future susy phase. In addition, only models with non-zero μ_0 allow a true susy breaking minimum in the absence of soft masses.

At our current level of analysis neglecting possible yukawa coupling changes, soft susy breaking terms and phases in the potential, the higgs structure is as in figure 1 so there is no exothermic transition to the exact susy minimum with EWSB.

If higgs studies at the Large Hadron Collider (LHC) confirm an extended higgs structure with non-zero v, one could be encouraged to imagine the rise of an exactly supersymmetric civilization in the distant future providing the Yukawa couplings decrease in the transition to exact susy or some extra source of energy input is found. On the other hand, if the LHC confirms the MSSM or an extended higgs model with $v = 0$ One might expect a structureless future susy universe with no atomic or molecular physics.

The mass squared matrix discussed above allows positive higgs masses even in the absence of soft breaking terms. Current limits on higgs masses will further constrain the viable parameter space although, because of the new interactions with the singlet higgs, one must avoid blindly applying the published limits on higgs masses. This is contrary to the case of the MSSM where soft higgs masses are essential. If, however, future experimental constraints require soft higgs masses even in the SESHM, there could be a parameter space region allowing an exothermic transition to an exact susy universe with EWSB. In principle, with sufficient experimentation at CERN, the effects of the
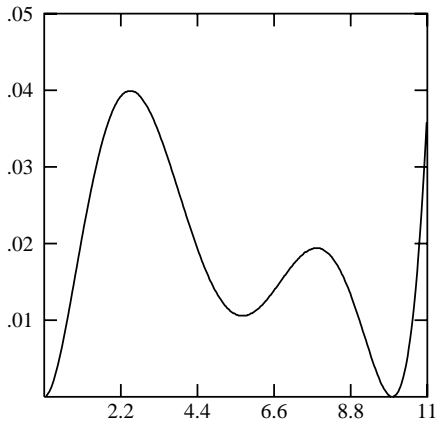
Fig. 1. The effective potential as a function of the magnitude of the doublet higgs field showing qualitatively the false vacuum of broken susy with EWSB and the two degenerate vacua of exact susy. Numerical values are given in arbitrary units.

soft breaking masses can be disentangled and the question of whether or not there could be an exothermic transition to the exact susy state with EWSB could be re-analysed. Another broken susy minimum with no EWSB might also exist corresponding to solution three discussed above.

The second susy ground state (solution 2), appearing at the origin in figure 1 has no EWSB. In this ground state, matter exists only in a permanently ionized state with no possibility of electrons condensing onto nuclei.

A question raised at this conference is whether this work is related to the anthropic principle (AP). The answer is no or, at least, not yet. Physical principles summarize a large body of experimental observation. A good example is Hamilton's principle of least action. The AP is the observation that, within narrow bounds, the laws of physics are such as to allow the evolution of intelligent life. Nothing in the large body of current experimental observations leading to the AP suggests that intelligent life will also arise in a future phase of the universe such as the exact susy phase discussed here. The anthropic principle cannot be used to predict the future. The fact that we exist does not prove that life forms of comparable (limited) intelligence will exist in the distant future. In the work reviewed here, there is no prediction as to whether or not the future susy phase will be friendly to advanced life forms. Instead we merely point out a possible correlation between parameters of the higgs potential measurable in the near future to possibilities for life or their absence in a distant future susy universe.

On the other hand, if intelligent life forms do evolve in some future atomphilic susy phase, those beings might be able to deduce that their existence implied a prior broken susy phase. For example, if the universe had been born in a susy phase of zero vacuum energy

there might not have been sufficient structure formation in the very early universe or the universe might have collapsed in the early instants. Also, we know that CP violation was crucial in the production of the baryon asymmetry which, in turn, seems essential to the development of stars, planets, and advanced life forms. There seems to be insufficent CP violation in the standard model so it is widely expected that susy CP violation played an essential role. Susy CP violation, however, is presently thought to come from the non-zero relative phases of the soft breaking terms and the μ parameter such as might exist in a broken susy phase but not in an exact susy phase. In the SESHM, as discussed above, there might be no soft breaking terms but CP violation in the broken susy phase could come from a non-zero relative phase of the vevs of S and H. These also would disappear in the exact susy phases (solutions 1 and 2 above) where $S_0 v_0 = 0$.

Subsequent to the work described here [6], a paper [10] has appeared applying similar inter-phase arguments to the EWSB phase transition of the early universe. There is at present a rapidly growing body of work studying other aspects of metastable susy breaking.

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