Landscape Implications of Extended Higgs Models

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Abstract

From several points of view it is strongly suggested that the current universe is unstable and will ultimately decay to one that is exactly supersymmetric (SUSY). The possibility that atoms and molecules form in this future universe requires that the degenerate electron/selectron mass is non-zero and hence that electroweak symmetry breaking (EWSB) survives the phase transition to exact SUSY. However, the minimal supersymmetric standard model (MSSM) and several of its extensions have no EWSB in the SUSY limit. Among the extended Higgs models that have been discussed one stands out in this regard. The Higgs sector that is revealed at the Large Hadron Collider (LHC) will therefore have implications for the future universe. We also address the question as to whether the transition to the exact SUSY phase with EWSB is exothermic.

Keywords: Supersymmetry; string landscape; SUSY phase transition; singlet extended SUSY Higgs models; LHC

1 Introduction

From the observation of a small positive vacuum energy in our world whereas its early stages had a much higher vacuum energy, from the persistent indications of supersymmetry with zero vacuum energy in the simplest manifestations of string theory and from the dynamical connection between worlds of differing vacuum energy in string theory, it seems quite likely that the present universe is intrinsically unstable. See for example Ref. [1]. Ultimately the

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universe should undergo a phase transition to a supersymmetric ground state. This final phase might be a supersymmetric anti-deSitter world of negative vacuum energy but, apart from predicting an ultimate big crunch, such a world would share many of the properties of the zero vacuum energy SUSY world discussed in this article. We do need to assume that there is no state of massively negative vacuum energy since the universe might rapidly fall into such an state, if available, and would then collapse on a microscopic time scale. This assumption is not inconsistent with experiment nor with any unique prediction of string theory.

Once in one of these SUSY states the universe should never return to a deSitter state [2]. Other things being equal, a universe that was supersymmetric from the beginning would probably not generate sufficient structure to give rise to life. For this reason or for others it has been stated [3] that life would be impossible in a SUSY universe. On the other hand, if an already evolved universe became supersymmetric through vacuum decay, it is possible, given favorable values of the parameters of the theory, that life could re-evolve in the SUSY background. The primary properties of such a SUSY universe where all particles are degenerate with their SUSY partners derive from a weakening of the Pauli exclusion principle [4]. It seems that atoms and molecules could exist in such a world [5] providing the common particle/sparticle masses in the SUSY phase were non-zero.

In the Schroedinger equation and in its relativistic Dirac counterpart, all atomic and molecular energy levels are proportional to the electron/selectron mass. In the limit of vanishing electroweak symmetry breaking (EWSB) atomic and molecular binding energies would, therefore, vanish while mean radii of atoms and molecules would tend to infinity. For instance, in the variational approach taken in Ref.[5], the ground state energy and mean radius of a system with Z SUSY protons and N SUSY electrons of mass m were found to be

$$E(N,Z) = -\frac{Nme^4}{2}\left(Z-\frac{5}{16}(N-1)\right)^2 \tag{1.1}$$

$$r(N,Z) = \frac{3}{2me^2}\left(Z-\frac{5}{16}(N-1)\right)^{-1} \quad . \tag{1.2}$$

One would expect that life would not arise in a world consisting only of elementary particles with no electromagnetic bound states.

From the point of view of the question as to whether post-transition life forms could exist, the problem arises that in the Minimal Supersymmetric Standard Model (MSSM) and in most of its extensions, electroweak symmetry breaking vanishes in the exact SUSY limit leaving all fermions massless as we shall discuss below.

Similarly if the atomic masses were greater in the exact SUSY phase than in the broken SUSY phase, energy conservation would require an endothermic phase transition in matter from the broken SUSY world to exact SUSY. However, the nucleon masses are dominated by non-perturbative QCD confinement effects and are much greater than the light quark masses. The masses of atoms are therefore somewhat insensitive to the masses of quarks. A small increase in the masses of SUSY neutrons, protons, and electrons could be compensated by the release of vacuum energy or by the release of excitation energy given that scalar particles are not bound by the Pauli principle. Of course, for isolated hydrogen or helium, this energy source is not available and it would be complicated to consider whether the required energy could come from nearby heavier atoms.

Therefore, for simplicity we ask if there is a theory where the Higgs vacuum expectation values in the broken SUSY phase are greater than or equal to those in the exact SUSY phase so that an exothermic transition to exact SUSY could lead in a simple way to a world supporting atoms and molecules.

Note that one cannot at present predict that a future SUSY universe would or would not be such as to support atomic and molecular binding. We can, however, point to a possible determination at the Large Hadron Collider (LHC) that the parameters of the Higgs potential are such as to suggest an answer to this question. The possibility of a scientific test distinguishes the question from a purely philosophical one.

Although we have derived some inspiration from string theory, this paper is exclusively an investigation of SUSY Higgs models where there are exact SUSY as well as possible broken SUSY minima. In these models there are no anti-deSitter vacua so, in the absence of any experimental constraints, we can ignore the question of whether in string theory there are local minima with massively negative vacuum energy.

In section II of the current paper we examine the MSSM and several of the extended SUSY Higgs models. We find that only one of these, the "nearly minimal supersymmetric standard model" (nMSSM) preserves EWSB in the SUSY limit thus allowing non-zero masses for the fermions and their degenerate superpartners.

In section III we discuss two models where spontaneous SUSY restoration occurs preserving EWSB. These models are analogous to other recently proposed models for meta-stable SUSY breaking [6] where the long lifetime of the current phase can be accomodated.

The meta-stability of the broken SUSY phase of the extended Higgs models was, in fact, noted decades ago by Fayet although one hoped then that this false vacuum could be made arbitrarily long lived. [7].

Finally, conclusions are presented in section IV.

2 MSSM and Extended Higgs Models in the Exact Susy Limit

The properties of several extended Higgs models have been the subject of much study in recent years following the pioneering work of Fayet [7]. They offer the promise of solving the μ problem and relieving stringent experimental constraints on the MSSM [8]. The field has been comprehensively reviewed [9] in the past year. For a recent summary of models with an additional Higgs singlets see Ref.[10].

The scalar potential of these models with broken SUSY consists of F terms derivable from a superpotential, W,

$$V_F = \sum_\phi \left| \frac{\partial W}{\partial \phi} \right|^2 \tag{2.1}$$

plus D terms and soft terms both of which break SUSY. The soft terms consist of scalar mass terms plus terms proportional to terms in the superpotential. The soft breaking terms are often assumed to be consequences of a SUSY breaking mechanism in a "hidden sector" which interacts gravitationally or otherwise with the particles of our world.

In the exact SUSY phase, if one exists, the soft terms will summarily drop to zero and the D terms will disappear at the potential minimum.

With sufficient experimentation, the LHC will be able to measure the magnitude of soft terms and subtract them out to determine whether the other parameters of the Lagrangian are such as to allow EWSB and therefore molecular physics in the SUSY limit. In this paper, therefore, we do not consider soft terms.

In this section we investigate the exact SUSY limit of the MSSM and several extended models. We drop terms in the scalar potential involving charged fields since these have no possible vacuum expectation values in a charge conserving theory. We find and investigate charge conserving minima without prejudice as to whether or not charge violating minima are possible.

2.1 MSSM

The minimal supersymmetric standard model is defined by a superpotential

$$W = \mu H_u \cdot H_d \quad , \tag{2.2}$$

the dot product of two Higgs doublets being defined by

$$H_u \cdot H_d = H_u^0 H_d^0 - H_u^+ H_d^- \quad . \tag{2.3}$$

Since we do not consider the possibility that the charged Higgs fields could acquire vacuum expectation values, we suppress any occurence of charged fields. Thus the dot product in 2.3 is taken to be equivalent to the product of the neutral fields.

The F term in the scalar potential of the MSSM, restricted to neutral fields, is

$$V_F = |\mu|^2 \left(|H_u|^2 + |H_d|^2 \right) \quad . \tag{2.4}$$

The D terms are

$$V_D = \frac{g_1^2 + g_2^2}{8} \left(|H_d|^2 - |H_u|^2 \right)^2 + \frac{g_2^2}{2} \left(|H_d|^2 |H_u|^2 - |H_u \cdot H_d|^2 \right)^2 \quad . \tag{2.5}$$

Since we restrict our attention to the potential of the neutral Higgs fields, we can discard the term in g_2^2.

The soft Higgs mass terms are

$$V_{soft} = m_d^2 \left| H_d \right|^2 + m_u^2 \left| H_u \right|^2 \quad . \tag{2.6}$$

In the absence of the soft mass terms, the minimum of the potential is at

$$< H_u >=< H_d >= 0 \quad . \tag{2.7}$$

The D terms vanish at this minimum but since the Higgs fields have zero vacuum expectation value, the electroweak symmetry breaking also vanishes. Thus, in the exact SUSY limit of the MSSM, there are no fermion masses and hence no electromagnetically bound atoms.

2.2 NMSSM

The next to minimal supersymmetric standard model (NMSSM) introduces a singlet superfield S. The superpotential is defined by

$$W = \lambda S H_u \cdot H_d + \frac{\kappa}{3} S^3 \quad . \tag{2.8}$$

The corresponding scalar potential, restricted to neutral fields is,

$$V_F = \left| \lambda S \right|^2 \left(\left| H_u \right|^2 + \left| H_d \right|^2 \right) + \left| \lambda H_u H_d + \kappa S^2 \right|^2 \quad . \tag{2.9}$$

The D terms are, as in the MSSM,

$$V_D = \frac{g_1^2 + g_2^2}{8} \left(\left| H_d \right|^2 - \left| H_u \right|^2 \right)^2 + \frac{g_2^2}{2} \left(\left| H_d \right|^2 \left| H_u \right|^2 - \left| H_u \cdot H_d \right|^2 \right)^2 \quad . \tag{2.10}$$

The soft terms are

$$V_{soft} = m_d^2 \left| H_d \right|^2 + m_u^2 \left| H_u \right|^2 + m_S^2 \left| S \right|^2 + \left(A_s \lambda S H_u \cdot H_d + \frac{\kappa}{3} A_\kappa S^3 + h.c. \right) \quad . \tag{2.11}$$

As the soft terms go to zero, the potential becomes symmetric in H_u and H_d so that

$$< H_u >=< H_d >= v_0 \quad . \tag{2.12}$$

This ensures that the D terms vanish at the minimum. The minimum also defines a vacuum expectation S_0 of the S field.

$$< S >= S_0 \quad . \tag{2.13}$$

Since the potential is positive definite or zero, any field configuration where the potential vanishes is necessarily a true minimum. Thus the configuration

$$S_0 = v_0 = 0 \tag{2.14}$$

is a true supersymmetric minimum. Since V_F is a sum of positive definite terms this is the only SUSY minimum and it fails to provide for EWSB.

2.3 UMSSM

This minimal SUSY model with an extra U(1) is defined by the superpotential

$$W = \lambda S H_u \cdot H_d \quad . \tag{2.15}$$

together with an extra U(1) gauge symmetry coupling to the two Higgs and to the S field with charges Q_u, Q_d, and Q_S. The F terms in the scalar potential are

$$V_F = \lambda^2 \left(|H_u H_d|^2 + |S|^2 \left(|H_u|^2 + |H_d|^2 \right) \right) \quad . \tag{2.16}$$

The D terms are as in the MSSM plus terms from the additional U(1):

$$V_D = V_{D,MSSM} + \frac{g_U^2}{2} \left(Q_{H_d} |H_d|^2 + Q_{H_u} |H_u|^2 + Q_S |S|^2 \right)^2 \quad . \tag{2.17}$$

In exact SUSY, the D terms must vanish at the minimum of the potential.

The symmetry of the superpotential implies that $Q_S + Q_{H_u} + Q_{H_d} = 0$. The vanishing of V_F and V_D at the SUSY minimum requires in any case that the positive definite term in $|H_u H_d|^2$ vanishes at the minimum and, therefore, $v_0 = 0$, i.e. no EWSB in the SUSY limit.

2.4 nMSSM

The nearly minimal supersymmetric standard model (nMSSM) is defined by the superpotential

$$W = \lambda S \left(H_u \cdot H_d - v^2 \right) \quad . \tag{2.18}$$

where, again, S is an electroweak singlet superfield.

The F terms in the scalar potential are then

$$V_F = \lambda^2 \left(S^2 (|H_u|^2 + |H_d|^2) + (|H_u H_d| - v^2)^2 \right) \quad . \tag{2.19}$$

The D terms are the same as in the MSSM in eq.2.5, vanishing at the generic potential minima:

$$< H_u > = < H_d > = v_0 \quad , \quad < S > = S_0 \quad . \tag{2.20}$$

The soft terms correspond to mass terms for the scalars plus terms proportional to terms in the superpotential. These soft terms, possibly governed by SUSY breaking in a hidden sector, vanish in the SUSY limit. In this exact SUSY limit, setting the soft terms to zero, the extrema are defined by

$$\left. \frac{\partial V}{\partial S} \right|_0 = 2 S_0 v_0^2 = 0 \tag{2.21}$$

and

$$\left.\frac{\partial V}{\partial H_u}\right|_0 = v_0(v_0^2 - v^2 + S_0^2) = 0 \quad . \tag{2.22}$$

In the SUSY limit the absolute minimum of the scalar potential is at

$$v_0 = v \quad , \quad S_0 = 0 \quad . \tag{2.23}$$

Thus for the nMSSM in the SUSY limit there is vanishing vacuum energy and with a broken electroweak symmetry ($v_0 \neq 0$).

All of the models of this section have been extensively studied in [7] and subsequent papers. It was, in fact, noted there that the nMSSM would allow an exact SUSY with EWSB. This observation acquires relevance in the current context of a final transition to the exact SUSY phase. In the following section we will study two toy models with a meta-stable broken SUSY minimum decaying to an exact SUSY with EWSB. These are both generalizations of the nMSSM (and the other extended Higgs models).

3 Meta-stable Models with EWSB in the Exact Susy Limit

In this section we consider first a model coupled through singlet Higgs fields to a mirror world (hidden sector) indicated by tildes and then secondly a model with no mirror symmetry.

3.1 meta-stable model with mirror symmetry

The Higgs superpotential is taken to be

$$W = \lambda\left(S(H_u \cdot H_d - v^2) + \tilde{S}(\tilde{H}_u \cdot \tilde{H}_d - v^2) + \mu_0 S\tilde{S}\right) \quad . \tag{3.1}$$

The mirror Higgs fields do not couple directly to normal matter and they may or may not be coupled to mirror matter. As we shall show, the corresponding scalar potential has minima with broken and unbroken SUSY. The neutral field F terms in the scalar potential take the form

$$\begin{aligned} V_F = \lambda^2 \Big(& \left|H_u H_d - v^2 + \mu_0 \tilde{S}\right|^2 + |S|^2 \left(|H_u|^2 + |H_d|^2\right) \\ & + \left|\tilde{H}_u \tilde{H}_d - v^2 + \mu_0 S\right|^2 + \left|\tilde{S}\right|^2 \left(\left|\tilde{H}_u\right|^2 + \left|\tilde{H}_d\right|^2\right)\Big) \quad . \end{aligned} \tag{3.2}$$

The D terms in the potential are as in the mirrored nMSSM.

$$\begin{aligned} V_D = & \frac{g_1^2 + g_2^2}{8}\left(|H_d|^2 - |H_u|^2\right)^2 + \frac{g_2^2}{2}\left(|H_d|^2 |H_u|^2 - |H_u H_d|^2\right) \\ & + \frac{g_1^2 + g_2^2}{8}\left(\left|\tilde{H}_d\right|^2 - \left|\tilde{H}_u\right|^2\right)^2 + \frac{g_2^2}{2}\left(\left|\tilde{H}_d\right|^2 \left|\tilde{H}_u\right|^2 - \left|\tilde{H}_u \tilde{H}_d\right|^2\right) \quad . \end{aligned} \tag{3.3}$$

The vacuum expectation values of the Higgs, to be determined by the minimization conditions, are

$$< H_u > = < H_d > = v_0 \tag{3.4}$$
$$< \tilde{H}_u > = < \tilde{H}_d > = \tilde{v}_0 \tag{3.5}$$
$$< S > = S_0 \tag{3.6}$$
$$< \tilde{S} > = \tilde{S}_0 \quad . \tag{3.7}$$

The minimization conditions are

$$\frac{1}{\lambda^2}\frac{\partial V_F}{\partial \tilde{S}}\bigg|_0 = 0 = \mu_0(v_0^2 - v^2 + \mu_0 \tilde{S}_0) + 2\tilde{v}_0^2 \tilde{S}_0 \tag{3.8}$$

$$\frac{1}{\lambda^2}\frac{\partial V_F}{\partial H_u}\bigg|_0 = 0 = v_0(v_0^2 - v^2 + \mu_0 \tilde{S}_0 + S_0^2) \tag{3.9}$$

$$\frac{1}{\lambda^2}\frac{\partial V_F}{\partial S}\bigg|_0 = 0 = \mu_0(\tilde{v}_0^2 - v^2 + \mu_0 S_0) + 2v_0^2 S_0 \tag{3.10}$$

$$\frac{1}{\lambda^2}\frac{\partial V_F}{\partial \tilde{H}_u}\bigg|_0 = 0 = \tilde{v}_0(\tilde{v}_0^2 - v^2 + \mu_0 S_0 + \tilde{S}_0^2) \tag{3.11}$$

The most obvious solution is
Solution 1: $v_0 = \tilde{v}_0 = v \quad , \quad S_0 = \tilde{S}_0 = 0$
This is the ground state of the model and corresponds to an exact supersymmetry (vanishing vacuum energy), and with EWSB ($v_0 \neq 0$).

A broken SUSY solution with, however, no EWSB, lies at $v_0 = \tilde{v}_0 = 0 \quad , \quad S_0 = \tilde{S}_0 = \frac{v^2}{\mu_0} \quad .$

A SUSY breaking minimum can be found with non-zero parameters $v_0, \tilde{v}_0, S_0, \tilde{S}_0$. We can see from eqs. 3.8,3.9,3.10, and 3.11 that

$$\tilde{S}_0 = \frac{\mu_0}{2\tilde{v}_0^2} S_0^2 \tag{3.12}$$
$$S_0 = \frac{\mu_0}{2v_0^2} \tilde{S}_0^2 \quad . \tag{3.13}$$

The solution with nonvanishing vevs for the two singlet Higgs are

$$S_0 = \frac{2v_0\tilde{v}_0}{\mu_0}\left(\frac{\tilde{v}_0}{v_0}\right)^{1/3} \tag{3.14}$$
$$\tilde{S}_0 = \frac{2v_0\tilde{v}_0}{\mu_0}\left(\frac{v_0}{\tilde{v}_0}\right)^{1/3} \quad . \tag{3.15}$$

Substituting these into the minimization conditions gives complementary cubic equations for v_0^2 and $\tilde{v}_0^2$:

$$(v_0^2 - v^2)^3 = -8(1 + \frac{2\tilde{v}_0^2}{\mu_0^2})^3 \tilde{v}_0^2 v_0^4 \tag{3.16}$$

$$(\tilde{v}_0^2 - v^2)^3 = -8(1 + \frac{2v_0^2}{\mu_0^2})^3 v_0^2 \tilde{v}_0^4 \quad . \tag{3.17}$$

These have the sole solution:

$$v_0^2 = \tilde{v}_0^2 = \frac{3\mu_0^2}{8}\left(\sqrt{1 + \frac{16v^2}{9\mu_0^2}} - 1\right) \quad . \tag{3.18}$$

The vev's have an upper limit

$$v_0 = \tilde{v}_0 = v/\sqrt{3} \quad . \tag{3.19}$$

Thus, the degenerate supermultiplets are heavier in the exact SUSY phase than the fermions in the broken SUSY phase. This would make the transition to exact SUSY endothermic.

3.2 A meta-stable Model without Mirror Symmetry

A simpler model without the mirror symmetry of the previous section is the following.

Consider the most general renormalizable Higgs superpotential with a pair of doublets and a single extra Higgs singlet supermultiplet [7]:

$$W = \lambda\left(S(H_u \cdot H_d - v^2) + \frac{\lambda'}{3}S^3 + \frac{\mu_0}{2}S^2\right) \quad . \tag{3.20}$$

If v and μ_0 are taken to vanish this is the superpotential of the $NMSSM$. If λ' and μ_0 are absent, this is the $nMSSM$. If all of λ', μ_0, and v vanish and an additional U(1) gauge interaction is introduced, this becomes the UMSSM.

Since, the superpotential of eq. 3.20 contains all the previously mentioned models as special cases, we refer to it simply as the Singlet Extended Susy Higgs Model (SESHM).

The F terms in the scalar potential are

$$V_F = \lambda^2\left(\left|H_u \cdot H_d - v^2 + \lambda' S^2 + \mu_0 S\right|^2 + |S|^2\left(|H_u|^2 + |H_d|^2\right)\right) \quad . \tag{3.21}$$

The D terms are as in the MSSM, eq. 2.5, vanishing at the minima of the potential. The soft terms are

$$\begin{aligned} V_{soft} =& m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 \\ &+ (A_s \lambda S H_u \cdot H_d + A_1 \lambda v S + A_2 \lambda \mu_0 S^2 + A_3 \lambda\lambda' S^3 + h.c.) \quad . \end{aligned} \tag{3.22}$$

It seems that little if any work has been done on models with a non-zero μ_0. This parameter can be set to zero, though with some loss of generality, by imposing a discrete symmetry under $S \to -S$ in the scalar potential. Note that, although continuous symmetries can also be seen in the superpotential, the scalar potential can exhibit additional discrete symmetries.

In this paper we will neglect the soft SUSY breaking terms. They will, in any case, vanish if SUSY breaking disappears but they would, if present, quantitatively affect the analysis of the broken SUSY phase and, therefore, of the inter-phase relationships that we are interested in.

The crucial point for the present paper is that a sufficiently detailed experimental study of the Higgs potential at the LHC can isolate the soft terms and determine whether the remaining terms are such as to allow EWSB in the SUSY limit.

Similarly, in the present analysis, we will ignore phases in the Higgs sector. In later work we will go beyond these toy models to incorporate phases and soft terms. Phases could be interesting from the point of view of CP violation and could also affect the inter-phase relationships.

Ignoring the soft Higgs mass terms, the symmetry of the scalar potential guarantees that the vevs of the two Higgs are equal and at this symmetry point the D terms vanish.

The conditions for an extremum of the scalar potential F terms are

$$\frac{1}{\lambda^2}\frac{\partial V_F}{\partial S}\bigg|_0 = 0 = (2\lambda' S_0 + \mu_0)(v_0^2 - v^2 + \mu_0 S_0 + \lambda' S_0^2) + 2v_0^2 S_0 \tag{3.23}$$

$$\frac{1}{\lambda^2}\frac{\partial V_F}{\partial H_u}\bigg|_0 = 0 = v_0(v_0^2 - v^2 + \mu_0 S_0 + (\lambda' + 1)S_0^2) \quad . \tag{3.24}$$

Since the potential is positive definite or zero, any localized solution of eqs. 3.23 and 3.24 with vanishing vacuum energy, $V_F(0)$, is necessarily a minimum.

The most obvious solution is
Solution 1: $v_0 = v \quad , \quad S_0 = 0$
This is one of the two grounds state of the model and corresponds to an exact supersymmetry (vanishing vacuum energy) with EWSB ($v_0 \neq 0$).

A second solution is
Solution 2: $v_0 = 0 \quad , \quad S_0 = \frac{-\mu_0 \pm \sqrt{\mu_0^2 + 4\lambda' v^2}}{2\lambda'}$
This solution is also supersymmetric with a vanishing vacuum energy at the minimum but with no EWSB.

A third solution with SUSY breaking but no EWSB is
Solution 3: $v_0 = 0 \quad , \quad S_0 = \frac{-\mu_0}{2\lambda'}$

If both v_0 and S_0 are non-zero, we find a **solution 4** with SUSY breaking plus EWSB.

The conditions become

$$2\lambda' S_0^2 + \mu_0 S_0 - 2v_0^2 = 0 \tag{3.25}$$

$$(\lambda' + 1)S_0^2 + \mu_0 S_0 + v_0^2 - v^2 = 0 \quad . \tag{3.26}$$

One can combine these conditions to find

$$(2\lambda' + 1)S_0^2 + \frac{3}{2}\mu_0 S_0 - v^2 = 0 \tag{3.27}$$

and

$$(\lambda' - 1)S_0^2 - (3v_0^2 - v^2) = 0 \quad . \tag{3.28}$$

This latter equation then predicts $v_0^2 > v^2/3$ if $\lambda' > 1$ and $v_0^2 < v^2/3$ if $\lambda' < 1$.

In solving equation 3.27 it is convenient to define the variable

$$z = \frac{16v^2(2\lambda' + 1)}{9\mu_0^2} \tag{3.29}$$

in terms of which

$$\mu_0 S_{0\pm} = -\frac{4v^2}{3} f_\pm(z) \tag{3.30}$$

with

$$f_\pm = (1 \pm \sqrt{1+z})/z \quad . \tag{3.31}$$

The functions $f_\pm(z)$ are shown in figure 1.

From eqs. 3.25 and 3.27 we have

$$v_0^2 = \frac{1}{2\lambda' + 1}(\lambda' v^2 - \mu_0 S_0(\lambda' - 1)/2) \quad . \tag{3.32}$$

Together these give S_0 and v_0 as a function of the parameters, λ' and μ_0.

In a future paper we would like to perform a complete analysis of the structure of minima and maxima of the scalar potential. Here we would only like to show that at least one true SUSY breaking minimum does exist.

The conditions for such an extremum to be a true minimum as opposed to a maximum or a saddle point is that all the eigenvalues of the mass squared matrix be positive. Neglecting phases, the mass squared matrix in the space of H_u, H_d, and S as obtained from the second derivatives of the scalar potential is

$$M^2 = \begin{vmatrix} \alpha + \zeta & \gamma - \zeta & \delta \\ \gamma - \zeta & \alpha + \zeta & \delta \\ \delta & \delta & \beta \end{vmatrix}$$

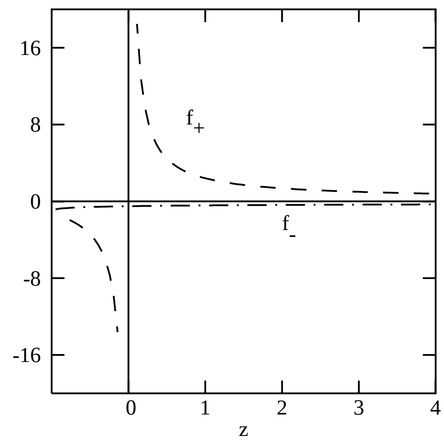

Figure 1: The function $f_+(z)$ (dashed line) and the function $f_-(z)$ (dot-dashed line) shown as functions of z. Both functions go to -1 at $z=-1$.

where

$$\alpha = \left.\frac{\partial^2 V_F}{\partial H_u^2}\right|_0 = 2\lambda^2(v_0^2 + S_0^2) \tag{3.33}$$

$$\gamma = \left.\frac{\partial^2 V_F}{\partial H_u \partial H_d}\right|_0 = 2\lambda^2(v_0^2 - S_0^2) \tag{3.34}$$

$$\delta = \left.\frac{\partial^2 V_F}{\partial H_u \partial S}\right|_0 = 2\lambda^2 v_0 \left(2(\lambda' + 1)S_0 + \mu_0\right) \tag{3.35}$$

$$\beta = \left.\frac{\partial^2 V_F}{\partial S^2}\right|_0 = 2\lambda^2 \left(6\lambda'^2 {S_0}^2 + 6\lambda' S_0 \mu_0 + 2\lambda'(v_0^2 - v^2) + \mu_0^2 + 2v_0^2\right) \tag{3.36}$$

and

$$\zeta = \left.\frac{\partial^2 V_D}{\partial H_u^2}\right|_0 = (g_1^2 + g_2^2)v_0^2 \quad . \tag{3.37}$$

The eigenvalues of the mass squared matrix satisfy

$$m_1^2 = 4\lambda^2 S_0^2 + 2\zeta \quad , \tag{3.38}$$

$$m_2^2 + m_3^2 = \alpha + \beta + \gamma \quad , \tag{3.39}$$

and

$$m_2^2 m_3^2 = \beta(\alpha+\gamma) - 2\delta^2 \quad . \tag{3.40}$$

The first squared mass is positive definite and corresponds to the eigenvector

$$\Psi_1 = \frac{H_u - H_d}{\sqrt{2}} \quad . \tag{3.41}$$

The positivity of m_2^2 and m_3^2 puts constraints on the parameter space of λ' and μ_0, namely

$$m_2^2 + m_3^2 = 2\lambda^2 \left(2\lambda' v^2 + \mu_0^2 + \mu_0 S_0(\lambda'+2)\right) > 0 \tag{3.42}$$

and

$$m_2^2 m_3^2 = -(2\lambda^2)^2 4 v_0^2 v^2 (1 + f_\pm(z)) > 0 \quad . \tag{3.43}$$

From eq. 3.43 and figure 1, one can see that positive Higgs masses requires negative z and the choice of the positive root f_+. This in turn requires that λ' be sufficiently negative. Numerically we find that we must have $\lambda' < -2$.

With the assumptions of the current paper (neglect of soft Higgs masses and neglect of phases) the Higgs potential is described by figure 2. Thus, if there is no change in yukawa couplings, the transition from the broken SUSY phase to the exact SUSY phase with EWSB is endothermic. In this paper we do not discuss solution three which, depending on the parameters may also be a true, local, SUSY-breaking minimum but one with no EWSB. Particle masses are products of Yukawa couplings and Higgs vevs. To relate the particle masses in separate phases we must know whether and by how much the yukawa couplings change in the transition. Such changes of couplings do not occur in the usual treatment of lagrangian Higgs models so we neglect them but they may occur in string theory (with presently undetermined magnitudes).

The value of the Higgs potential at the broken SUSY minimum is

$$V_F(0) = \lambda^2 S_0^2 (S_0^2 + 2v_0^2) \quad . \tag{3.44}$$

$V_F(0)$ gives a contribution to the dark energy in the broken SUSY phase. Other contributions come from the mass splitting of fermions and sfermions and, perhaps, from compactification and thermal effects. The algebraic sum of these various contributions could be equated with the measured vacuum energy.

4 Conclusions

The primary result of the current paper is that, neglecting phases, a true broken SUSY minimum exists over a finite region of the parameter space without assuming the presence of additional soft mass terms. This requires a non-zero μ_0 parameter which was not part

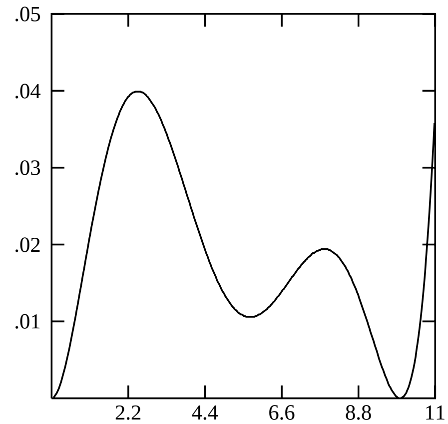

Figure 2: The effective potential as a function of the magnitude of the doublet Higgs field showing qualitatively the false vacuum of broken SUSY with EWSB and the two degenerate vacua of exact SUSY. The numerical scales are given in arbitrary units. The extremum with broken SUSY but without EWSB is not shown.

of previous analyses. Furthermore, while the MSSM, the NMSSM, and the UMSSM do not support a phase transition to an exact SUSY with non-zero fermion masses (EWSB), the SESHM with non-zero v does have EWSB as required for atomic and molecular structure in the SUSY phase. Thus, if the LHC finds a non-zero v in the scalar potential, the possibility is open that SUSY atoms and molecules will form after the expected phase transition to exact SUSY although, at present, it seems that an additional energy input would be required to permit the transition to proceed.

We present two toy models which have a meta-stable broken SUSY phase and an exact SUSY phase with EWSB. In these models one or more singlet Higgs with vacuum quantum numbers play the role of an inflaton field mediating the transition between phases of differing vacuum energy.

A second SUSY ground state (solution 2) has no EWSB. In this ground state, matter would be permanently ionized with no possibility of atomic structure.

It is not within the scope of the current paper to examine the phenomenology of the broken SUSY phase (solution 4) beyond the basic requirements discussed above so much remains to be done. Previous work without allowing for the contribution of the μ_0 parameter has been discussed in the reviews [9][10] of the extended Higgs models but it remains to be seen whether the currently allowed parameter space or that to be revealed in future LHC experiments is consistent with $v > 0$. By measuring the λ, λ', and μ_0 parameters and the Higgs masses, the LHC can also determine whether soft Higgs masses are needed in the broken SUSY phase.

At the present level of analysis, neglecting phases, soft SUSY breaking terms, and the possibility of the yukawa couplings decreasing in the transition, the common particle/sparticle mass in the exact SUSY phase is greater than the particle mass in the broken SUSY phase. Thus the transition would require extra energy input to proceed. This energy could come from the energy (primarily nuclear) stored in the Pauli towers. Otherwise, the transition would necessarily be to the SUSY phase without EWSB (solution 2). An interesting question for future study would be whether further extending the Higgs potential with additional Higgs multiplets or considering non-zero phases in the vev's could allow a broken SUSY phase with $v_0 > v$.

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