

Metastable Aspects of Singlet Extended Higgs Models

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Abstract

It has long been known that the broken supersymmetric (susy) phase of the singlet extended susy higgs model (SESHM) is at best metastable and the ground states of the model have vanishing vacuum energy and are exactly supersymmetric. If the SESHM is confirmed at the Large Hadron Collider (LHC), the numerical values of the parameters of the model have a bearing on key properties of the susy phase and might provide an estimate of the remaining time before a possible decay of our false vacuum. We provide some analysis of the model including a treatment of phases in the potential and soft higgs masses.

1 Introduction

In the minimal supersymmetric standard model (MSSM), the higgs sector consists of two higgs doublets one coupling to the up-type quarks and the other to the down-type quarks. This can be extended by the addition of a higgs singlet and such models have been the subject of extensive study over the last three decades as chronicled in the reviews [1, 2, 3]. The most general, renormalizable, singlet extended, susy higgs model (SESHM) corresponds to the superpotential

$$W = \lambda \left(S(H_u \cdot H_d - v^2) + \frac{\lambda'}{3} S^3 + \frac{\mu_0}{2} S^2 \right) \quad (1.1)$$

where S is the singlet higgs field. The dot product is defined as

$$H_u \cdot H_d = H_u^0 H_d^0 - H_u^+ H_d^- \quad . \quad (1.2)$$

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In the present work, focusing on the vacuum properties of the higgs potential, we suppress the charged higgs fields assuming, as usual, that they do not acquire vacuum expectation values.

This model was first written by Fayet [4] in the seventies and its metastability was already noted then. The vacuum expectation value of the S field corresponds to the μ parameter of the MSSM and the introduction of the singlet higgs therefore provides a possible dynamical solution to the so-called μ problem of the MSSM [5]. Most, if not all, of the subsequent phenomenological analysis has restricted the most general model by putting one or more of the parameters in 1.1 to zero [3]. For instance the “next to minimal susy standard model” (NMSSM) takes v and μ_0 to zero. The “UMSSM” takes v , λ' , and μ_0 all to zero and introduces an additional U(1) gauge symmetry. The “nearly minimal susy standard model”, (nMSSM) takes μ_0 and λ' to zero. All of these well studied limiting cases of the SESHM treat the model in the absence of the μ_0 coupling. The vanishing of μ_0 can be obtained, with some loss of generality, by requiring that the superpotential is odd under $S \rightarrow -S$ while the scalar potential is even. In addition, phenomenological studies have naturally concentrated on the broken susy phase whereas the properties of the exact susy ground state and the inter-phase relations of the model have been largely ignored.

With the accumulation of experimental evidence that the universe has a small positive vacuum energy and originally, in the inflationary era, had a much larger vacuum energy, interest in metastable models with a susy ground state has increased [6, 7, 8]. In string theory also, it is thought that the universe makes transitions among a large number of local minima in a string landscape. The original and primary manifestations of string theory had zero vacuum energy and were exactly supersymmetric as in the ground states of the SESHM. However, string theory also has a prominent manifestation with negative vacuum energy, the anti-deSitter universe of the AdS/CFT correspondence, and many other possible local minima with a wide variety of vacuum energies. If the universe were to fall into a state of large negative vacuum energy it would collapse on a short time scale in a “big crunch”. This leads to possible physical and philosophical problems which are irrelevant to this paper since we treat exclusively the SESHM. Thus, although string theory can be a source of inspiration, we do not feel bound by specific string possibilities in advance of experimental confirmation.

In the absence of soft susy breaking, the SESHM has a positive definite scalar potential and, therefore, no negative vacuum energy solutions. We assume that the soft breaking terms preserve this feature in order to provide a model for the current phase of the universe. Another difference between string theory and lagrangian higgs models is that in the latter the parameters of the higgs potential are the same in each local minimum although the vevs of the fields could differ widely. In string theory the parameters of the potential are also thought to be minimum dependent.

In a recent paper [9] we have shown that the SESHM has four critical points where all derivatives with respect to the fields vanish. These are:

Solution 1: Exact Susy with Electroweak Symmetry Breaking (EWSB)

Solution 2: Exact Susy with no EWSB

Solution 3: Broken Susy with no EWSB

Solution 4: Broken Susy with EWSB

Solution 4, is, obviously, most close to our universe. A susy ground state with EWSB will, in general, support non-zero particle masses and therefore susy atoms and molecules [10]. If such a susy ground state exists, it is interesting to consider [11] whether susy life forms might evolve in a future exact susy universe. Solution 2, on the other hand, corresponds to a supersymmetric plasma of elementary particles with no atomic, molecular, or condensed matter physics.

We are primarily interested in metastability aspects of the SESHM which have not been treated in earlier studies. It is theoretically possible to identify which regions of the parameter space have a doublet higgs vacuum expectation value greater in solution 4 than in solution 1. Since quark and lepton masses are controlled by the doublet higgs vev, in matter dominated regions of configuration space, these regions of parameter space would have greater energy in solution 4 than in solution 1. This would allow an exothermic transition from our universe to that of solution 1 whereas, in other regions of parameter space there would be only an endothermic transition to solution 1 neglecting the small amount of vacuum energy that would be released. In either case there could be an exothermic transition to solution 2. The situation is schematically indicated in figure 1 projected into a one dimensional higgs space.

Of course, nucleon masses are proportional to the Λ parameter of QCD and are much larger than the corresponding quark masses. Thus, among the atomic constituents, only the lepton masses are sensitive to the higgs vev. Since in normal matter even the electron energy density is much greater than the vacuum energy density, the transition to exact susy would require extra energy input if the higgs vev in the exact susy minimum were greater than that in our broken susy minimum.

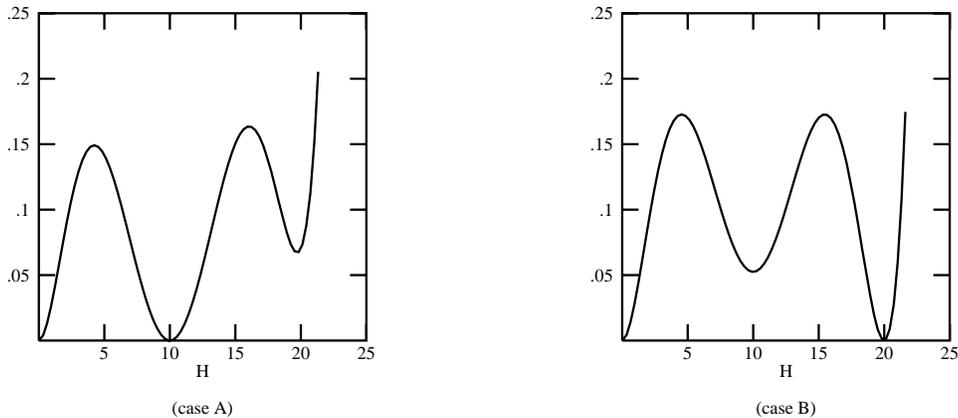


Figure 1: The higgs potential is plotted schematically versus a one-dimensional higgs field in the exothermic case A and the endothermic case B. The minimum corresponding to solution 3 is not shown. See text.

Although we cannot, a priori, predict which region of parameter space is realized in

nature, the issue can be experimentally resolved by a sufficiently detailed study of the higgs sector at the LHC or some future accelerator. We cannot even be sure, of course, that the higgs structure of nature is that of the SESHM but this also will soon be experimentally determined. Thus the question is one of physics and not merely one of philosophy. In string theory, on the other hand, it is not clear at present whether there are any unique predictions that are falsifiable in advance of the next transition.

In ref.[9] we treated solution 4 in a simplified toy model ignoring phases and soft susy breaking terms. In the current article we give the phase analysis. This will eventually allow a treatment of CP violation from the higgs sector of the SESHM although we do not proceed here to a phenomenological analysis. We also give a partial analysis of soft terms.

There is, at present, no good theory of susy breaking in our universe nor is there a good theory for why the vacuum energy is as small as it is. In the MSSM and in other phenomenological approaches one usually adds to the scalar potential explicit “soft” susy breaking masses and A terms as discussed in section 2. These are often postulated to derive from susy breaking in some “hidden” sector communicated to our sector through gravitational or other interactions but they could also be thought of as an effective representation of a more complete model of susy breaking. In the toy model of ref.[9] ignoring phases it was possible to find consistent higgs sector solutions without treating soft terms. In the current treatment of complex fields and complex Lagrangian parameters we find that it is no longer theoretically consistent to ignore soft terms in the broken susy phase. The current suggestion, then, is that the susy-breaking critical points of the SESHM are promoted to true minima by non-perturbative effects assumed here to be parametrizable in terms of soft higgs masses.

The crucial points are:

- 1) When the transition to exact susy occurs, simultaneously in both sectors, these soft terms will vanish and
- 2) A sufficiently detailed experimental analysis of the broken susy phase can separate the soft terms from the other terms in the higgs potential and determine whether the transition to the susy solution 1 is exothermic or not.

A further question that can be answered when all the parameters of the higgs sector are experimentally determined is what will be the expected lifetime of the current universe. If the higgs vevs in the broken susy state approach those of the exact susy state or if the height of the barrier goes to zero the lifetime of the broken susy phase will go to zero. It is, therefore, of interest to ask for which regions in parameter space, if any, do these conditions prevail. Experiments at the LHC may thus be relevant to the expected lifetime of the current universe. Of course, the fact that the current universe has survived for some 13.7 billion years also constrains the parameters and suggests that the expected future lifetime should not be orders of magnitude less than this.

In section II we review the SESHM and the critical point conditions on the parameters and vevs. A full analysis of the soft terms is postponed to a time when the LHC has determined that a singlet higgs is, in fact, present. To illustrate the suggested procedure it is sufficient to consider a soft higgs scalar mass squared M_{ξ}^2 and a common soft doublet

higgs mass squared M_h^2 . Allowing differing soft doublet masses is an inessential complication to the questions we are pursuing in the present article although such a mass difference will clearly perturb the quantitative correlations we present in later sections. The necessary corrections can be made once the soft masses are experimentally determined. Similarly, small radiative corrections to our results should be expected. In section III we examine the parameter space as a function of these two soft squared masses taken to be positive. In fact, however, there are no theoretical or experimental reasons for these soft squared masses to be positive providing the physical higgs squared masses are non-negative and consistent with experimental constraints.

In section IV we consider the possibility that M_S^2 and M_h^2 are negative which allows a (fine-tuned) situation in which the higgs vacuum energy in the broken susy phase is equal to the experimental dark energy. We note that there are, at present, no successful models for the dark energy which do not involve fine tuning even if the anthropic principle is used to trivially “predict” that a theory with a small dark energy exists. There are also, however, other possible contributions to the dark energy such as from compactification [12], loop contributions with broken susy or, generically, any physical mechanism which induces an essential difference between susy partners such as thermal effects [13].

Thus, while we find the possibility of negative M_S^2 interesting to consider, its exact value cannot be firmly predicted by this method unless or until the other contributions to the dark energy are known to be small and the other parameters of the higgs potential are known.

Our results are briefly summarized in section V.

We rely on the experimental fact that the vacuum energy is positive. We exclude, therefore, contradictory regions of parameter space with large negative values of M_S^2 or M_H^2 . This has the consequence that the susy minima are lower in energy density than the broken susy minima and the broken susy minima are metastable as in ref. [4]. If experiments had found a negative vacuum energy in the broken susy phase, it would be possible to write a SESHM with a broken susy ground state. These points and the entire current paper assume that we can neglect non-perturbative effects except as effectively describable in terms of the soft terms. This assumption is common to the MSSM and most, if not all, of the phenomenological treatments of the SESHM.

2 The Singlet Extended Susy Higgs Model

The superpotential of the SESHM given in eq. 1.1 leads to the F term contributions to the scalar potential

$$V_F = \lambda^2 \left(|H_u \cdot H_d - v^2 + \lambda' S^2 + \mu_0 S|^2 + |S|^2 (|H_u|^2 + |H_d|^2) \right) \quad . \quad (2.1)$$

In addition we have the gauge generated D terms

$$V_D = \frac{g_1^2 + g_2^2}{8} (|H_d|^2 - |H_u|^2)^2 + \frac{g_2^2}{2} (|H_d|^2 |H_u|^2 - |H_u \cdot H_d|^2)^2 \quad (2.2)$$

where g_1 and g_2 are the $U(1)$ and $SU(2)$ gauge coupling constants. Since we restrict our attention to the potential of the neutral higgs fields, we can discard the second term in g_2^2 .

The soft susy breaking terms are

$$V_{soft} = \lambda^2 \left(m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + (A_s S H_u \cdot H_d + A_1 v S + A_2 \mu_0 S^2 + A_3 \lambda' S^3 + h.c.) \right) \quad . \quad (2.3)$$

The full potential is the sum of these three terms

$$V = V_F + V_D + V_{soft} \quad . \quad (2.4)$$

Since the soft parameters are, a priori, undetermined we can, without loss of generality, factor out a λ^2 as in eq. 2.3. A result of this convention is that the critical point conditions do not depend on λ although all the mass squared eigenstates are proportional to λ^2 apart from small calculable D term contributions.

The critical points of the potential are those points in field space where all first derivatives with respect to the fields vanish. The critical points are true local minima if the eigenvalues of the higgs mass squared matrix are all non-negative.

In the exact susy phase, the soft terms are absent as discussed in the introduction. The symmetry of the remaining potential under $H_u - H_d$ interchange, implies that at the minima of the potential the two doublet higgs will have equal vacuum expectation values and the D terms will vanish which is, in fact, a condition for exact susy.

The most general renormalizable model including soft terms corresponds to a complicated multi-parameter potential whose full analysis will only be justified when and if singlet higgs fields are experimentally discovered. In this paper we show how certain questions related to metastability can be addressed while restricting our attention to soft mass terms only, with the additional simplification of equal soft H_u and H_d mass squared terms. This latter restriction leads to equal doublet higgs vevs in the susy breaking minima and therefore to the absence of D terms at the minima. The D terms will however contribute to the mass squared matrix and must therefore be taken into account.

We analyse, therefore, the simplified model

$$V = V_F + V_D + \lambda^2 m_S^2 |S|^2 + \lambda^2 m_H^2 (|H_u|^2 + |H_d|^2) \quad . \quad (2.5)$$

Hermiticity requires that λ , m_S^2 , and m_H^2 be real. Without loss of generality we can take one further parameter real; we choose v real. The phase dependence of the potential then corresponds only to the phases of the remaining three terms in V_F so the phase of λ' can be absorbed into that of the field S and the parameter μ_0 , λ' then being taken real. Apart from the overall factor of λ^2 the Lagrangian is then dependent on the the complex parameter μ_0 and on the real parameters m_S^2 , m_H^2 , λ' and v , a total of six real parameters. As a function of these we seek points in the field space

$$\begin{aligned} \langle S \rangle &= S_0 \\ \langle H_u \rangle &= \langle H_d \rangle = v_0 \end{aligned} \quad (2.6)$$

at which all the first derivatives of V vanish. The critical point conditions are, therefore,

$$\frac{1}{\lambda^2} \frac{\partial V}{\partial S^\dagger} \Big|_0 = 0 = (2\lambda' S_0^* + \mu_0^*)(v_0^2 - v^2 + \lambda' S_0^2 + \mu_0 S_0) + S_0(2|v_0|^2 + m_S^2) \quad (2.7)$$

and

$$\frac{1}{\lambda^2} \frac{\partial V}{\partial H_u^\dagger} \Big|_0 = 0 = v_0^*(v_0^2 - v^2 + \lambda' S_0^2 + \mu_0 S_0) + v_0(|S_0|^2 + m_H^2) \quad . \quad (2.8)$$

These two equations determine possible values of the complex vevs, S_0 and v_0 in terms of the parameters of the potential. It is convenient, however, to invert the relations and consider μ_0 and v as determined by the vevs S_0 and v_0 .

The higgs contribution to the vacuum energy is

$$V(0) = \lambda^2 \left(|v_0^2 - v^2 + \lambda' S_0^2 + \mu_0 S_0|^2 + 2|S_0|^2 |v_0|^2 + m_S^2 |S_0|^2 + 2|v_0|^2 m_H^2 \right) \quad . \quad (2.9)$$

The critical points are independent of λ while the higgs vacuum energy is proportional to λ^2 but this cannot be made arbitrarily small without similarly reducing the masses of the physical higgs eigenstates unless the soft masses m_S^2 and m_H^2 are correspondingly fine-tuned. We will set λ to unity with the understanding that the physical higgs masses can be scaled by a common factor of λ while the vacuum energy density is scaled by a factor of λ^4 .

The critical points with exact susy are found by solving eqs.2.7 and 2.8 neglecting the soft mass terms as discussed in the introduction. Two degenerate solutions are found with zero vacuum energy:

Solution 1: $v_0 = v$, $S_0 = 0$: exact susy with EWSB

Solution 2: $v_0 = 0$, $\lambda' S_0^2 + \mu_0 S_0 - v^2 = 0$: exact susy; no EWSB

Since, neglecting soft terms, the scalar potential is positive definite or zero, any localized critical points with zero vacuum energy are true minima.

Two more solutions can be found with non-zero vacuum energy and non-zero singlet vev S_0 . For these, to illustrate the procedure, we keep the soft mass terms as representative of all the soft terms . If a singlet higgs is confirmed at LHC it will be of interest to do the complete global analysis. We emphasize, therefore, that the current paper does not attempt to provide tight phenomenological fits to the LEP constraints. It is known that the existence of a singlet higgs significantly loosens the constraints on the MSSM [3] even in the absence of a non-zero μ_0 parameter.

Solution 3 has no EWSB ($v_0 = 0$) and a cubic equation for $|S_0|$:

$$(2\lambda' S_0^* + \mu_0^*)(-v^2 + \lambda' S_0^2 + \mu_0 S_0) + S_0 m_S^2 = 0 \quad . \quad (2.10)$$

For this to be a true local minimum we would have to require that all the physical higgs squared masses are non-negative here. We will not pursue this question at solution 3 in the present paper although it might ultimately become interesting to consider whether a transition from solution 3 to solution 4 could describe an EWSB transition in the early universe.

Here we concentrate on the parameter space conditions that will make solution 4 a true local minimum. Solution 4, which has EWSB, corresponds to the extensive phenomenological discussion in the literature [2, 3] treating various limiting cases of the SESHM. In solution 4, we take the experimental EWSB to require $|v_0| \approx 175$ GeV.

With non-zero v_0 and S_0 , the two equations 2.7 and 2.8 can be combined to yield

$$-\frac{v_0^*}{v_0}(2\lambda'S_0 + \mu_0)(|S_0|^2 + m_H^2) + S_0^*(2|v_0|^2 + m_S^2) = 0 \quad (2.11)$$

and

$$v^2 = v^{*2} = v_0^2 + \lambda'S_0^2 + \mu_0 S_0 + \frac{v_0}{v_0^*}(|S_0|^2 + m_H^2) \quad . \quad (2.12)$$

The vacuum energy in solution 4, $V_4(0)$, is given by substituting the vevs v_0 and S_0 satisfying eqs. 2.11 and 2.12 into eq. 2.9.

$$V_4(0) = \lambda^2 \left(|S_0|^4 + |S_0|^2(2|v_0|^2 + m_S^2 + 2m_H^2) + m_H^4 + 2m_H^2|v_0|^2 \right) \quad . \quad (2.13)$$

It is convenient to define the real dimensionless variables

$$A = |v_0/S_0| \quad , \quad (2.14)$$

$$B = (2|v_0|^2 + m_S^2)/|S_0|^2 \quad , \quad (2.15)$$

and

$$C = 1 + m_H^2/|S_0|^2 \quad . \quad (2.16)$$

Rather than deal immediately with the cubic equation for S_0 , we prefer to define the parameter μ_0 in terms of the value for S_0 assumed to run over all consistent values. From eqs. 2.11 and 2.12:

$$\mu_0 = \frac{S_0^* B v_0}{v_0^* C} - 2\lambda'S_0 \quad . \quad (2.17)$$

We could, of course, investigate the $\mu_0 = 0$ case as in the well-studied limiting cases of the SESHM but we prefer to maintain generality.

3 The Higgs Mass Squared Matrix

The higgs mass squared matrix in solution 4 is obtained by taking the scalar potential of eq. 2.5 and expanding around the vevs of the neutral fields. We write

$$\begin{aligned} S &= s + S_0 \\ H_u &= h_u + v_0 \\ H_d &= h_d + v_0 \end{aligned} \quad (3.1)$$

Taking v real, we can write the four complex numbers as

$$\begin{aligned}\mu_0 &= |\mu_0| e^{i\phi_\mu} \\ S_0 &= |S_0| e^{i\phi} \\ v_0 &= |v_0| e^{i\phi_0} \quad .\end{aligned}\tag{3.2}$$

It is convenient to use the symmetric and antisymmetric combinations of the h fields:

$$\begin{aligned}h_u &= h_p + h_m \\ h_d &= h_p - h_m \quad .\end{aligned}\tag{3.3}$$

Using eqs. 2.11 and 2.12 to eliminate the terms linear in the shifted fields and discarding terms cubic and quartic in the fields, we have

$$V = V_4(0) + X \quad .\tag{3.4}$$

The mass squared matrix can be written

$$X = \Psi_1^2 M_1^2 / 2 + \Psi_2^2 M_2^2 / 2 + \Delta X \quad .\tag{3.5}$$

The two real fields Ψ_1 and Ψ_2 have been trivially separated out with the definitions

$$\begin{aligned}\Psi_1 &= h_m e^{-i\phi_0} + h_m^\dagger e^{i\phi_0} \\ \Psi_2 &= h_m e^{-i\phi_0} / i - h_m^\dagger e^{+i\phi_0} / i\end{aligned}\tag{3.6}$$

and the corresponding squared masses

$$\begin{aligned}M_1^2 &= 2C|S_0|^2 + 8(g_1^2 + g_2^2)|v_0|^2 \\ M_2^2 &= 0 \quad .\end{aligned}\tag{3.7}$$

The massless field is absorbed into a massive Z boson. The remaining scalar boson can be quite light depending on the values of C and $|S_0|^2$. The possibility of observing such a light higgs boson of the NMSSM in the radiative decay of the Υ has been recently discussed [14].

The remaining mass squared matrix is

$$\begin{aligned}\Delta X &= -h_p^2 C |S_0|^2 v_0^* / v_0 + 2h_p h_p^\dagger (2|v_0|^2 + C|S_0|^2) + 2h_p s S_0^* v_0^* \\ &\quad + 2h_p s^\dagger S_0 v_0^* (B/C + 1) - h_p^{\dagger 2} C |S_0|^2 v_0 / v_0^* + 2h_p^\dagger s S_0^* v_0 (B/C + 1) + 2h_p^\dagger s^\dagger S_0 v_0 \\ &\quad - s^2 C |S_0|^2 \lambda' v_0^* / v_0 + s s^\dagger |S_0|^2 B (B/C^2 + 1) - s^{\dagger 2} C |S_0|^2 \lambda' v_0 / v_0^* \quad .\end{aligned}\tag{3.8}$$

This can be simplified and written in terms of real fields, Φ_i , by the transformations:

$$\begin{aligned}h_p &= (\Phi_1 + i\Phi_2) e^{i\phi_0} \\ h_p^\dagger &= (\Phi_1 - i\Phi_2) e^{-i\phi_0}\end{aligned}\tag{3.9}$$

$$\begin{aligned}s &= (\Phi_3 + i\Phi_4) e^{i\phi_0} \\ s^\dagger &= (\Phi_3 - i\Phi_4) e^{-i\phi_0} \quad .\end{aligned}\tag{3.10}$$

Then

$$\begin{aligned}
\Delta X = & 4\Phi_1^2|v_0|^2 + 4\Phi_2^2(|v_0|^2 + C|S_0|^2) + \Phi_3^2|S_0|^2(B^2/C^2 + B - 2C|\lambda'| \cos(\phi_\lambda)) \\
& + \Phi_4^2|S_0|^2(B^2/C^2 + B + 2C\lambda') \\
& + 4\Phi_1\Phi_3|S_0|^2 A \cos(\phi - \phi_0)(B/C + 2) + 4\Phi_1\Phi_4|S_0|^2 A \sin(\phi - \phi_0)(B/C + 2) \\
& - 4\Phi_2\Phi_3|S_0|^2 AB \sin(\phi - \phi_0) + 4\Phi_2\Phi_4|S_0|^2 AB \cos(\phi - \phi_0) \quad .
\end{aligned} \tag{3.11}$$

Next we make the transformations:

$$\begin{aligned}
\Phi_3 &= X_3 \cos(\phi - \phi_0) - X_4 \sin(\phi - \phi_0) \\
\Phi_4 &= X_3 \sin(\phi - \phi_0) + X_4 \cos(\phi - \phi_0) \\
\Phi_1 &= X_1 \\
\Phi_2 &= X_2
\end{aligned} \tag{3.12}$$

after which

$$\begin{aligned}
\Delta X = & 4X_1X_1|v_0|^2 + 4X_1X_3|v_0|^2(B/C + 2) + 4X_2X_2|v_0|^2(1 + C/A^2) + 4X_2X_4|v_0|^2B/C \\
& + X_3X_3|S_0|^2(B^2/C^2 + B - 2C\lambda' \cos(\beta)) + 4X_3X_4|S_0|^2C\lambda' \sin(\beta) \\
& + X_4X_4|S_0|^2(B^2/C^2 + B + 2C\lambda' \cos(\beta)) \quad .
\end{aligned} \tag{3.13}$$

where we have put

$$\beta = 2(\phi - \phi_0) \quad . \tag{3.14}$$

We scan over six real parameters, λ' , m_S^2 , m_H^2 , $|S_0|, v$, and ϕ . If λ' vanishes as in the nMSSM, it is possible to diagonalize the mass matrix analytically. We prefer, however, to maintain greater generality so we numerically solve for the eigenvectors and the mass squared eigenvalues using the Jacobi method [15]. We require that all the eigenvalues are non-negative and examine the resulting solution space. In the numerical study we take $|v_0|$, nominally 175 GeV, as our unit of energy. In these units the exothermic transition to solution 1 discussed above requires that v be less than unity (i.e. by v we mean $v/|v_0|$).

From eq. 2.17 we have

$$|\mu_0| = |S_0| \left(B^2/C^2 + 4\lambda'^2 - 4\lambda'(B/C) \cos(\beta) \right)^{1/2} \quad . \tag{3.15}$$

From eqs. 2.11, 2.12, and 2.17

$$v^2 = e^{2i\phi_0} \left(|v_0|^2 - \lambda'|S_0|^2 e^{i\beta} + |S_0|^2(B/C + C) \right) \quad . \tag{3.16}$$

The condition that v is real determines the phase ϕ_0 in terms of five real parameters:

$$\tan(2\phi_0) = \frac{\lambda'|S_0|^2 \sin(\beta)}{|v_0|^2 + |S_0|^2(B/C + C) - \lambda'|S_0|^2 \cos(\beta)} \tag{3.17}$$

after which

$$v^2 = \cos(2\phi_0)(|v_0|^2 + |S_0|^2(B/C + C)) - \lambda'|S_0|^2 \cos(2\phi_0 + \beta) \tag{3.18}$$

and

$$\phi = \beta/2 + \phi_0 \quad . \quad (3.19)$$

The phase of μ_0 is determined by the reality of v^2 :

$$\tan(\phi_\mu) = \frac{\tan(2\phi_0 - \phi)(B/C - 2|\lambda'| \cos(\beta))}{B/C - 2\lambda'(\cos(\beta) + \sin(\beta) \tan(2\phi_0 - \phi))} \quad . \quad (3.20)$$

We now scan over the six free parameters requiring that the squared higgs masses be positive and recording the consistent values of the parameters.

4 Solutions with positive soft squared masses

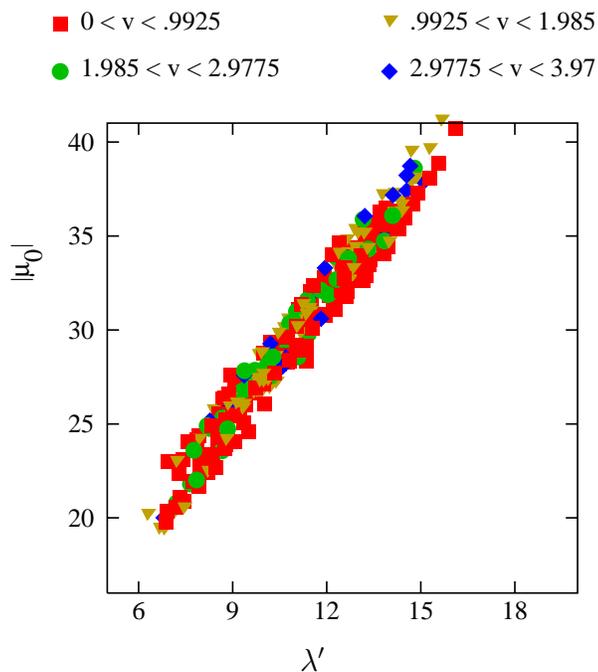


Figure 2: The space of solutions for λ' and $|\mu_0|$ with the corresponding values of v color-shape coded in the case of positively constrained soft squared masses.

In this section we constrain the soft squared masses to be positive. We scan for solutions with the physical Higgs squared masses between 0.1 and 10 in units of $|v_0|^2$, $0 < v < 4$, and soft squared masses between 0 and 7.5. This first requirement is equivalent to the assumption that the minimum higgs mass is greater than about 57 GeV although, as noted above and in ref. [14], in the SESHM, the experimental limits need to be carefully re-examined and a very light higgs is possible. Radiative corrections to the masses proportional to the electroweak couplings could also perturb the masses.

	minimum	maximum	mean	std dev
λ'	5.93	16.1	11.0	2.19
M_3	<u>0.32</u>	1.17	0.55	0.15
M_4	<u>0.32</u>	4.09	1.34	0.46
M_5	6.19	<u>10.0</u>	9.06	0.76
M_6	0.34	7.98	5.80	0.76
$V_4(0)$	3.0	4.0	3.7	0.21
$ S_0 $	0.96	1.11	1.05	0.03
M_S^2	2.37	<u>7.27</u>	5.53	0.84
M_H^2	0.0	0.29	.074	.058
v	<u>0.0</u>	<u>4.0</u>	1.2	0.90
$ \mu_0 $	16.9	41.0	29.8	4.75
ϕ	0.0	6.283	3.14	1.85
ϕ_0	2.36	5.49	4.03	0.88
ϕ_μ	0.00	6.283	3.11	2.54
β	2.75	3.52	3.14	.097
M_{min}	<u>0.32</u>	0.94	0.52	0.14

Table 1: Allowed values of parameters requiring positive physical higgs squared masses and positive soft squared masses in solution 4. M_3 through M_6 together with M_1 give the range of physical higgs masses. Underlined quantities are somewhat arbitrarily imposed. Other quantities are consequent limits of the solution space. Subject to these constraints, the minimum, maximum, mean, and rms deviations from the mean are tabulated. M_{min} gives the minimum value of M_1 and M_3 through M_6 . Dimensional quantities are given in terms of the doublet higgs vev $v_0 = 175$ GeV.

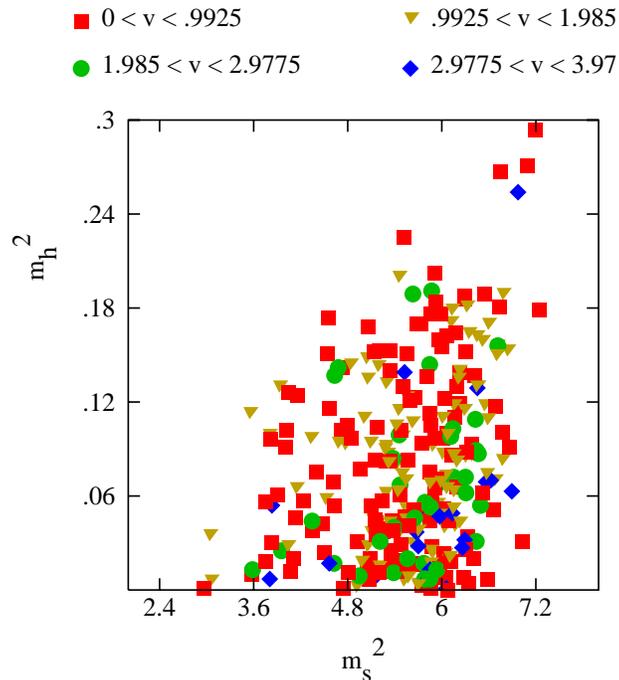


Figure 3: The space of solutions for positive soft squared masses m_s^2 and m_h^2 with the corresponding values of v color-shape coded.

As noted in the introduction, no solutions are found with both soft masses vanishing although copious solutions are found with vanishing m_H^2 . The vacuum energy density at the minimum lies between 2.9 and 4.0 in units of $|v_0|^4$. This agrees with the expectation that the vacuum energy is of order of the higgs vev to the fourth power. It is of course far greater than the observed value of the vacuum energy in our broken susy state.

A distinctive property of this solution space is that λ' and $|\mu_0|$ are in the non-perturbative region as can be seen in figure 2. This is similar to the finding of large couplings in another model of the metastable vacuum [7]. Similarly, the required soft mass m_S is quite large as seen in figure 3 although solutions are found with small m_H .

5 Solutions with negative soft squared masses

As discussed in the introduction, the soft squared masses can be negative as long as the physical higgs particles have positive squared masses, a condition we can impose in the scan over parameters. The freedom to consider negative soft squared masses can be used to parameterize the fine tuning of the vacuum energy that is, at present, necessary to understand its smallness relative to the natural scale of v_0^4 . Ignoring other possible contributions we may equate $V_4(0)$ with the observed vacuum energy [16]:

$$V_4(0) = (5.9 \pm 0.2)meV^4 = (6.36 \pm 0.16) \cdot 10^{-57}|v_0|^4 \quad (5.1)$$

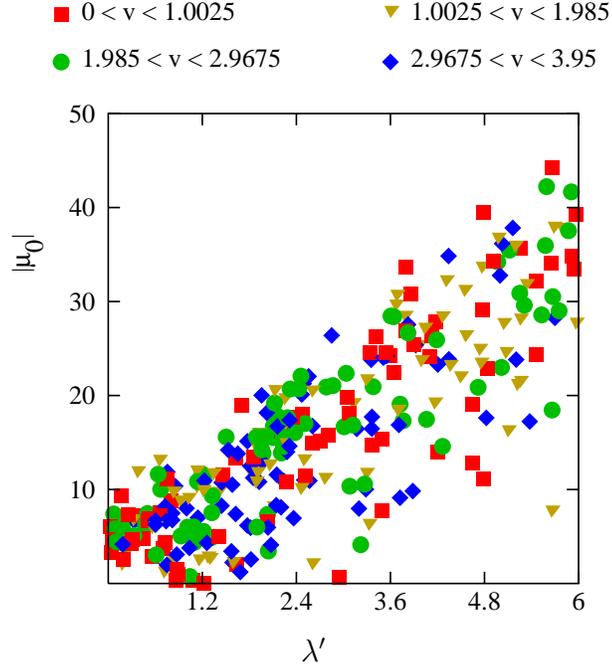


Figure 4: The space of solutions for λ' and $|\mu_0|$ with the corresponding values of v color-shape coded allowing for negative squared soft masses and requiring that the higgs vacuum energy be equal to the observed value of dark energy.

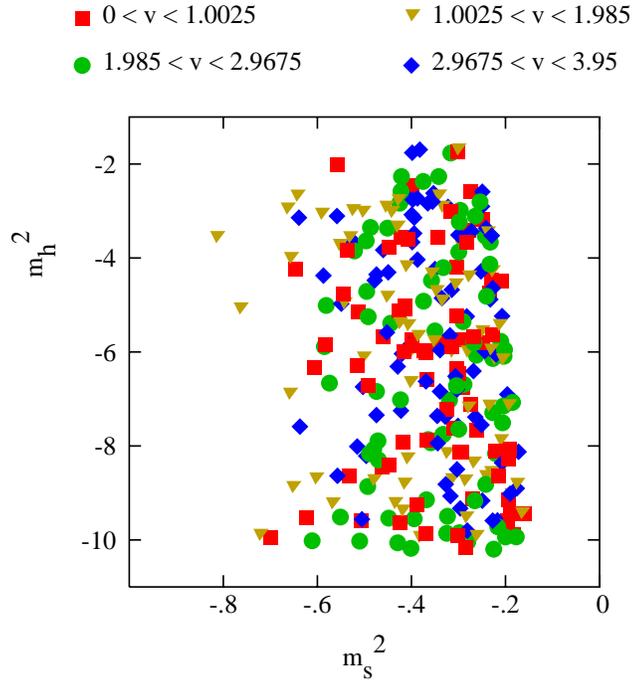


Figure 5: The space of solutions for m_s^2 and m_h^2 is shown with the corresponding values of v color-shape coded allowing for negative squared soft masses and requiring that the higgs vacuum energy be equal to the observed dark energy.

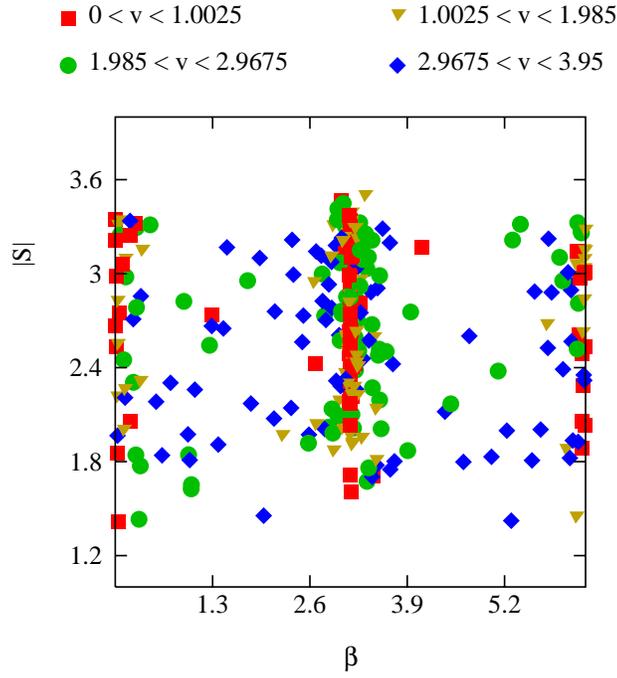


Figure 6: The space of solutions for β and $|S_0|$ with the corresponding values of v color-shape coded allowing for negative squared soft masses and requiring that the higgs vacuum energy be equal to the observed dark energy. It is seen that the low values of v are concentrated at $\beta = n\pi$

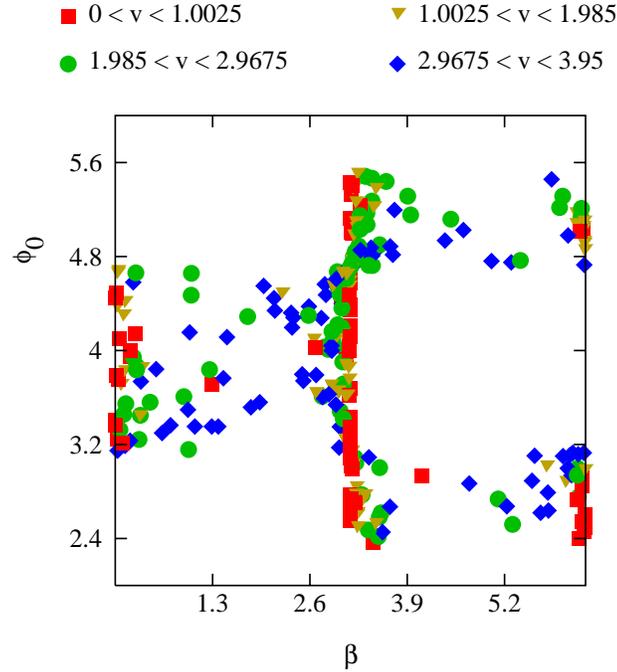


Figure 7: The space of solutions for β and ϕ_0 with the corresponding values of v color-shape coded allowing for negative squared soft masses and requiring that the higgs vacuum energy be equal to the observed dark energy.

	minimum	maximum	mean	std dev
λ'	0.010	5.99	2.70	1.66
M_3	<u>0.33</u>	1.72	1.23	0.33
M_4	1.58	3.55	2.59	0.37
M_5	3.34	<u>9.93</u>	6.11	1.57
M_6	<u>0.327</u>	<u>9.75</u>	5.44	2.21
$V_4(0)$	<u>$6.2 \cdot 10^{-57}$</u>	<u>$6.52 \cdot 10^{-57}$</u>	<u>$6.36 \cdot 10^{-57}$</u>	<u>$0.16 \cdot 10^{-57}$</u>
$ S_0 $	1.37	3.48	2.64	0.52
M_S^2	-0.85	-0.16	-0.37	0.13
M_H^2	-10.2	-1.62	-6.38	2.40
v	<u>0.0</u>	<u>4.0</u>	2.01	1.14
$ \mu_0 $	0.05	45.2	15.9	10.2
ϕ	0.0	6.283	3.20	1.76
ϕ_0	2.36	5.48	3.96	0.88
ϕ_μ	0.00	6.283	3.21	2.29
β	0.00	6.283	3.08	1.76
M_{min}	<u>0.33</u>	1.61	1.09	0.26
C	-0.06	0.26	0.13	0.04

Table 2: Allowed values of parameters requiring that the higgs vacuum energy in solution 4 be equated with the observed vacuum energy. This requires that both of the soft squared masses are negative although the physical higgs squared masses are all required to be positive. M_3 through M_6 together with M_1 give the range of physical higgs masses. Underlined quantities are imposed. Other quantities are consequent limits of the solution space. M_{min} gives the overall minimum value of M_1 and M_3 through M_6 .

We may fix $V_4(0)$ at this value by choosing in eq. 2.13

$$m_S^2 = -C^2|S_0|^2 - 2|v_0|^2C + |v_0|^4 \frac{1}{\lambda^2|S_0|^2} \cdot 10^{-57}(6.36 + 0.16(2r - 1)) \quad (5.2)$$

where r is a random number between 0 and 1. It is interesting that in this part of parameter space it is possible to find solutions with small values of the couplings λ' and $|\mu_0|$ as shown in figure 4. thus suggesting a perturbative treatment of the higgs interactions. Some scatter plots of these solutions are shown in figs. 4,5,6, and 7. The correlation between values of m_S^2 and m_H^2 requiring both to be negative is shown in figure 5. It can be noted that, in this full theory with complex fields and the vev of the higgs potential near the observed dark energy, no solutions are found for the phase of the doublet higgs vev, ϕ_0 , near 0 or 2π . Solutions with ϕ_0 near π , however, could have small CP violation. A concentration of solutions near $\beta = n\pi$ can be seen in figure 6. In the limit of small μ_0 , this concentration becomes sharp and there is no CP violation in the higgs potential. The hypercube containing all the solutions is tabulated in table 2.

6 Summary

We have examined the singlet extended higgs model from the point of view of a possible transition from our world to an exactly supersymmetric world with electroweak symmetry breaking. We have included the effects of the quadratic superpotential term in the scalar higgs field which has been neglected in the analyses of other authors. Copious parameter space solutions have been found in which an exothermic transition to an exact susy world with EWSB is possible as well as many others where this is not possible. LEP experiments, eg. [17], leave large regions of parameter space unexplored especially for low $\tan\beta$. Experiments at the LHC will be able to decide which solutions are realized in nature if, in fact, a susy higgs structure with an extra singlet is confirmed. This can be done by examining the Higgs mixing term after the other parameters, λ' , μ_0 , and S_0 have been determined. In the most general model this term is

$$V_F = H_u \cdot H_d \left(v_0^2 - v^2 + \lambda' S_0^2 + \mu_0 S_0 \right)^\dagger + h.c. \quad (6.1)$$

This relation, of course, receives corrections proportional to electroweak gauge couplings and to differences of soft doublet Higgs masses. We have begun the analysis of phases in the potential which could eventually be of interest for CP violation. We have noted and explored physical regions of parameter space where the soft squared higgs masses are negative although the physical higgs squared masses are positive. Here we have fixed the vev of the higgs potential to the one sigma range of the observed dark energy. Allowing a somewhat wider range would lead to negligibly small corrections to the soft masses and other parameters. It must be remembered, of course, that there are other contributions to the dark energy as noted earlier. The point of this exercise is that large physical higgs masses could be made consistent with a small vacuum energy by taking negative soft mass squareds. In a future paper we would like to explore solution 3 which has exact susy but no EWSB. It

will be of interest to ask whether some or all of the physical regions of parameter space for solution 4 will also lead to a true minimum of the potential at the critical point of solution 3. In this case we will be able to consider whether the model is consistent with an EWSB transition from solution 3 to solution 4 occurring in the early universe.

We emphasize again that it is not the purpose of the present paper to provide tight phenomenological constraints for the parameters but merely to illustrate the existence of solutions with $v > 1$ as well as with $v < 1$ and to suggest that the experimental determination of which region is chosen by nature might have interesting physical consequences. Phenomenological predictions of the model for production mechanisms and decay chains at the LHC as well as the tightening of current experimental constraints are left for consideration in later papers with special attention to the effects of a non-zero μ_0 parameter.

Acknowledgements

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