Finite Matter Resolution of the Cosmological Entropy Problem

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Abstract

We discuss how entropy bounds, which are not respected in the standard cosmology, constrain the parameters of a previously suggested cosmology with a finite total mass [1],[2]. In that alternative cosmology the matter density was postulated to be a spatial delta function at the time of the big bang thereafter diffusing rapidly outward with constant total mass. Also discussed here are some related issues including the cosmic onion question, the information content of the universe, and the question of whether light trapping regions exist on a cosmic scale.

keywords: Cosmology, Bekenstein Entropy bound, Finite Mass

1 Introduction

The Bekenstein [3] bound on entropy can be formulated as stating that no object of radius r can have an entropy greater than the entropy of a black hole of that Schwarzschild radius. The bound was originally derived in the Gedenkenexperiment in which a body of given entropy is lowered into a black hole of slightly greater mass. It can, perhaps, be most easily seen by noting that any object that can collapse into a black hole due to gravity or any other force must begin with an entropy less than that of the black hole since the entropy in any process should increase or remain constant. The standard homogeneous and isotropic cosmology violates the Bekenstein bound since the entropy in a sphere of radius r grows

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as r^3 inevitably exceeding the entropy of a black hole of that horizon radius which grows as r^2. Thus this cosmology, although consistent with Einstein's equations, is inconsistent with thermodynamics if a dark energy transition to a big-crunch with negative cosmological constant is physically possible and, in any case, is inconsistent with the Bekenstein bound.

The entropy of a black hole of Schwarzschild radius r satisfies

$$\frac{S_{BH}(r)}{2\pi k} = \frac{r^2}{2{L_{Pl}}^2} \tag{1.1}$$

where the Planck length is

$$L_{Pl} = \sqrt{\hbar G/c^3} = 1.6 \cdot 10^{-35}\, m. \tag{1.2}$$

2 Bekenstein bound in a finite mass universe

If the entropy of any object of radius r is $S(r)$, the Bekenstein bound requires

$$S(r) < S_{BH}(r) \tag{2.3}$$

Known astrophysical objects including main sequence stars, white dwarfs, and neutron stars easily satisfy this bound but the standard homogeneous cosmology, inevitably violates the Bekenstein bound for large enough r as stated above. Various attempts to deal with the problem within the context of the infinite homogeneous universe have been summarized in ref. [4].

The entropy density of an object of density ρ, pressure p, and chemical potential μ at temperature T satisfies [5]

$$\frac{s(r)}{2\pi k} = \frac{1}{2\pi kT}\left(\rho(r) + p(r) - \mu\, n(r)\right) \tag{2.4}$$

where ρ is the energy density, $p(r)$ is the pressure, and $n(r)$ is the number density of constituents. The chemical potential μ is a mass factor for the constituents. Clearly, if the universe is infinite and homogeneous and the entropy density follows this equation, the total entropy in a sphere of radius r grows as r^3 violating the Bekenstein bound. This has motivated several alternative entropy bounds, the holographic entropy bound [6],[7], a covariant entropy bound [8], and a causal entropy bound [9]. All of these alternative bounds involve heuristic and unexplained assumptions.

Dark energy does not contribute to the entropy since $\mu = 0$ and $p = -\rho$. For a photon gas the chemical potential also vanishes and the pressure density is $\rho/3$ so that

$$\frac{s_\gamma(r)}{2\pi k} = \frac{2}{3\pi}\,\frac{1}{k\,T}\rho(r) = \frac{2}{3\pi kT} < E_\gamma > n_\gamma(r) \quad . \tag{2.5}$$

Over a wide range of temperatures the number density of photons is known to be proportional to the number density of baryons:

$$n_\gamma(r) \approx \frac{10^{10}}{6.05} n_b(r) \quad . \tag{2.6}$$

The average photon energy in a black body of photons is

$$< E_\gamma > \approx 2.701 \cdot kT \quad . \tag{2.7}$$

If we integrate up to radius r, the entropy, $S(r)$, in a sphere of that radius is therefore related to the total number of baryons, $N_b(r)$ in that sphere.

$$\frac{S_\gamma(r)}{2\pi k} \approx \frac{5.40}{3\pi} \cdot \frac{10^{10}}{6.05} N_b(r) \quad . \tag{2.8}$$

The large photon to baryon ratio implies that we can safely neglect the entropy carried by baryonic matter. In the standard homogeneous cosmology $N_b(r)$ grows as r^3 so that the total entropy from eq. 2.8 inevitably exceeds the entropy of a black hole of that Schwarzschild radius as alluded to above.

Clearly, a possible resolution is the construction of a model where the number of baryons in a sphere of radius r saturates or grows no more than quadratically with r. Two such models where, in fact, this number approaches a constant were proposed in ref.[1, 2].

In the first, a matter density with infinite range but finite total mass took the form

$$\rho(r,t) = \frac{M}{\pi^{3/2} R^3 a(t)^3} \, e^{-r^2/(Ra(t))^2} \quad . \tag{2.9}$$

Here R is a scale of inhomogeneity and $a(t)$ is approximately the scale factor of the Friedmann-Robertson-Walker model since in regions of low curvature Hubble's law can be derived [1]:

$$\vec{v} = \frac{\dot{a}}{a} \vec{r} = H\vec{r} \quad . \tag{2.10}$$

$a(t)$ is taken to vanish at $t = 0$, increase monotonically, and equal unity at present time.

In the second, a matter density with finite range, finite total mass, and a bubble topology took the form

$$\rho(r,t) = \frac{M}{5.15 R^3 a(t)^3} \, (3 - 2(r/(a(t)R)^2) \, e^{-r^2/(Ra(t))^2} \, \theta(3 - 2(r/(a(t)R)^2) \quad . \tag{2.11}$$

Both were obtained as the time-time component of the Einstein tensor for appropriately defined space-time metrics. In the present work we ignore possible effects of the off-diagonal components of the Einstein tensor taking the above densities as our starting point. The baryon number density is obtained in each case by dividing by the proton mass, m_p. Clearly, if we integrate over a sphere of sufficiently large radius each model is consistent with the Bekenstein bound since the enclosed entropy asymptotes to a constant. It could however, be asked whether for smaller radius some constraint on the parameters M or R is obtained.

For simplicity we restrict our analysis here to the first model although the second model will not lead to qualitatively different results.

The analysis is most appropriately done in the co-moving frame defined by the co-moving coordinate

$$r_c = r/a(t) \quad . \tag{2.12}$$

In this frame the matter density takes the time independent form

$$\rho_c(r_c) = \frac{M}{(R\sqrt{\pi})^3}\, e^{-{r_c}^2/R^2} \quad . \tag{2.13}$$

As $R \to \infty$ with M/R^3 constant, the model approaches the standard homogeneous cosmology. The number of baryons in a sphere of radius r_c is

$$\begin{aligned} N_b(r_c) &= \frac{M}{m_p(R\sqrt{\pi})^3} \int_0^{r_c} 4\pi {r_c'}^2 dr_c'\, e^{-(r_c'/R)^2} \\ &= \frac{2M}{m_p\sqrt{\pi}}\, \gamma(3/2, {r_c}^2/R^2) \end{aligned} \tag{2.14}$$

where γ is the incomplete Γ function of the given arguments.

This can be expressed in terms of the proton density and the co-moving matter density near $r = 0$ from eq. 2.13 or the current density near $r = 0$ in the expanding frame

$$\rho_c(0) = \rho(0, now) = \frac{M}{(R\sqrt{\pi})^3} \quad . \tag{2.15}$$

The density of the proton using the $R_p = 1$ fm proton radius is

$$\rho_p = \frac{3m_p}{4\pi R_p^3} \quad . \tag{2.16}$$

Thus we have

$$N_b(r_c) = \frac{3R^3}{2{R_p}^3} \frac{\rho_c(0)}{\rho_p}\, \gamma(3/2, {r_c}^2/R^2) \quad . \tag{2.17}$$

Observationally, $\rho_c(0)/\rho_p \approx 2\pi\, 10^{-45}$ so

$$N_b(r_c) = \frac{3\pi \cdot 10^{-45} R^3}{{R_p}^3}\, \gamma(3/2, {r_c}^2/R^2) \tag{2.18}$$

It might also be convenient to express the proton radius in terms of a particular combination of Planck lengths and Hubble lengths

$$
{R_p}^3 \approx \frac{1}{35.1} L_H {L_{Pl}}^2 \quad . \tag{2.19}
$$

Collecting factors it is seen that, in the co-moving frame, the entropy contained in a sphere of radius r_c is specified by

$$
\frac{S_\gamma(r_c)}{2\pi k} \approx 3.13 \cdot 10^{-34} \frac{R^3}{L_H {L_{Pl}}^2} \gamma(3/2, {r_c}^2/R^2) \tag{2.20}
$$

Relative to the entropy in a black hole of Schwarzschild radius r_c from eq. 1.1 this would be

$$
\frac{S_\gamma(r_c)}{S_{BH}(r_c)} \approx 6.27 \cdot 10^{-34} \frac{R}{L_H} \frac{R^2}{{r_c}^2} \gamma(3/2, {r_c}^2/R^2) \quad . \tag{2.21}
$$

Consistency with the Bekenstein bound is obtained if this ratio is less than unity for all r_c. The function

$$
\frac{1}{x^2}\gamma(3/2, x^2) = \frac{1}{x^2} \int_0^{x^2} dy \sqrt{y} e^{-y} \tag{2.22}
$$

clearly vanishes at $x = 0$ and $x = \infty$. It has a maximum value of about 0.379 at $x \approx 0.968$. Thus, consistency with the bound requires

$$
2.38 \cdot 10^{-34} \frac{R}{L_H} < 1 \tag{2.23}
$$

or

$$
R < 5.8 \cdot 10^{59} m \quad . \tag{2.24}
$$

If $R \to \infty$ in eq. 2.21 with fixed local density one recovers the standard cosmological matter density which violates the Bekenstein bound. The matter inhomogeneity implied by a finite value of R is usually thought to be constrained by the isotropy of the Cosmic Background Radiation (CBR) and by the homogeneity of the type Ia Supernovae. However, the CBR in general implies only that $R \geq 10^5 L_H$ and, in the case that the Milky Way is near the $r = 0$ position, a much smaller value of R is possible. The supernovae data do not extend much beyond redshift $z = 1$ so they do not strongly constrain R. In fact, indications from galaxy clusters suggest [10] a value [2] of R near the Hubble length, L_H. If we treat the Beckwith result as a lower limit on the R parameter, this and the Bekenstein bound imply the constraints

$$
0.94 < \frac{R}{L_H} < 0.42\, 10^{34} \tag{2.25}
$$

The upper limit on R from the entropy bound is not very restrictive but the fact that there is a finite upper limit at all is of interest. Even this very weak upper limit allows the estimate that there is a negligible probability to find two identical unrelated humans in the universe. To see this assume there is no more than one human civilization per solar system with 10^{11} humans in each, 10^{11} solar systems per galaxy and 10^{11} galaxies per Hubble length. Then the number of humans in the finite matter universe is no more than $10^{33}\,(R/L_H)^3$. Since this is much less than the number of distinct human genomes, $10^{3.6\,10^9}$, the probability of producing identical twins by random shuffling of genes is negligible.

The total information content in the universe is

$$I = \frac{S}{k \log 2} \tag{2.26}$$

This is infinite in the standard cosmology but finite and dependent on R in the finite matter cosmology. This has a bearing on the "cosmic onion" question in physics: Are there a finite number of constituent species and fundamental interactions or, in the onion analogy, is there no limit to the new fundamental physics that can be revealed as we peel off successive layers? If the total information content in the universe is finite as in the finite matter model, there cannot be an infinite number of constituent species or fundamental gauge groups. Thus, in this model, physics is finite at the fundamental level although there is still great scope for applied physics if and when the complexity of the fundamental laws is exhausted.

3 Constraint from absence of light trapping

A tighter bound on R can be obtained if one postulates that the total matter inside any radius r_c in the co-moving frame is less than the Schwarzschild mass corresponding to that horizon length. Then the universe will never form a large scale light trapping region. The total matter inside r_c is from eq. 2.17

$$M_b(r_c) = m_p N_b(r_c) = \frac{3 m_p R^3}{2{R_p}^3} \frac{\rho_c(0)}{\rho_p}\, \gamma(3/2, {r_c}^2/R^2) \quad . \tag{3.27}$$

In terms of the proton mass and radius the mass of a black hole with horizon r_c is

$$M_{BH}(r_c) = 7.95 \cdot 10^{37} m_p \frac{r_c}{R_p} \tag{3.28}$$

so

$$\frac{M_b(r_c)}{M_{BH}(r_c)} \approx 4\pi\, 10^{-83} (\frac{R}{R_p})^2 \frac{R}{r_c} \gamma(3/2, (r_c/R)^2) \quad . \tag{3.29}$$

The incomplete gamma function is such that for any R/r_c

$$\frac{R}{r_c} \gamma(3/2, (r_c/R)^2) < 0.466 \quad . \tag{3.30}$$

Thus, for any co-moving radius, r_c the mass enclosed is always less than a black hole of that horizon size if

$$R < 1.05\, L_H \quad . \tag{3.31}$$

Combining this no-cosmic-light-trapping requirement with the result of [10] leads to tight upper and lower bounds on the inhomogeneity of the universe

$$0.94\, L_H < R < 1.05\, L_H \quad . \tag{3.32}$$

Providing consistency with the Bekenstein entropy bound might be the clearest indication to-date that there exists no more than a finite amount of matter. Other successes such as the avoidance of infinite replication of each individual, the avoidance of infinite numbers of monsters in the multiverse and the avoidance of the measure problem of standard cosmology are, perhaps, more philosophical in nature as is the no-cosmic-light-trapping requirement. Nevertheless, it is a satisfying result of the entropy bound that the upper limit on the R parameter found here assures that there is a negligible probability to produce two identical but unrelated human beings anywhere in the universe.

Yet to be explored in the finite matter model are the strange astrophysical correlations that have been found [11] and the possibility of specifying the initial state in greater detail and deducing the consequences thereof. For instance, instead of attempting to predict the baryon asymmetry from CP violation, could one relate manifestations of CP violation to a CP asymmetry in the initial state? The answer might be "no" but the possibility does not exist in the standard cosmology with its pair production of baryons at the end of inflation.

All of this does not, of course, minimize the work that still needs to be done in the finite matter model to carry over the successes of the standard cosmology in fitting the length of the inflationary era and the primordial fluctuations that determine the anisotropies in the cosmic background radiation.

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